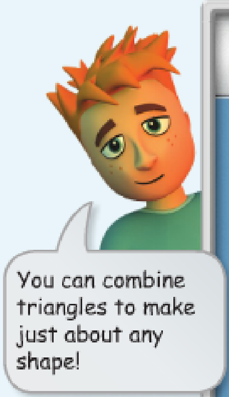
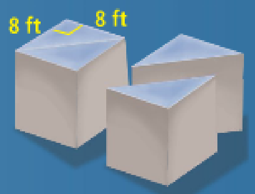
10-1 Areas of Parallelograms and Triangles

Objective To find the area of parallelograms and triangles


You can combine triangles to make just about any shape!

Solve It! **Getting Ready!**

A stage is being set up for a concert at the arena. The stage is made up of blocks with tops that are congruent right triangles. The tops of two of the blocks, when put together, make an 8 ft-by-8 ft square. The band has requested that the stage be arranged to form the shape of an arrow. Draw a diagram that shows how the stage could be laid out in the shape of an arrow with an area of at least 1000 ft^2 but no more than 1400 ft^2.



Dynamic Activity
Area of Parallelograms and Triangles

Lesson Vocabulary
- base of a parallelogram
- altitude of a parallelogram
- height of a parallelogram
- base of a triangle
- height of a triangle

Essential Understanding You can find the area of a parallelogram or a triangle when you know the length of its base and its height.

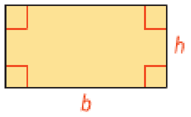
A parallelogram with the same base and height as a rectangle has the same area as the rectangle.

take note

Theorem 10-1 Area of a Rectangle

The area of a rectangle is the product of its base and height.

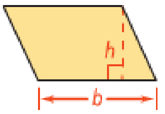
$A = bh$



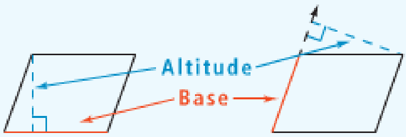
Theorem 10-2 Area of a Parallelogram

The area of a parallelogram is the product of a base and the corresponding height.

$A = bh$



A **base of a parallelogram** can be any one of its sides. The corresponding **altitude** is a segment perpendicular to the line containing that base, drawn from the side opposite the base. The **height** is the length of an altitude.



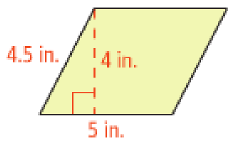
Problem 1 Finding the Area of a Parallelogram

Think

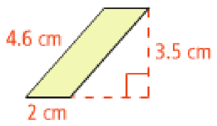
Why aren't the sides of the parallelogram considered altitudes?
Altitudes must be perpendicular to the bases. Unless the parallelogram is also a rectangle, the sides are not perpendicular to the bases.

What is the area of each parallelogram?

A



B



You are given each height. Choose the corresponding side to use as the base.

$A = bh$

$= 5(4) = 20$ Substitute for *b* and *h*.

The area is 20 in.^{2}.

$A = bh$

$= 2(3.5) = 7$

The area is 7 cm^2.

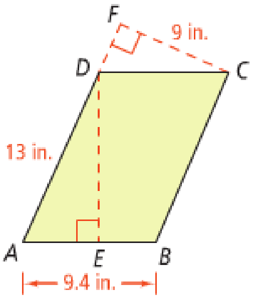
Got It? **1.** What is the area of a parallelogram with base length 12 m and height 9 m?

Problem 2 Finding a Missing Dimension

Think

What does $\overline{CF}$ represent?
$\overline{CF}$ is an altitude of the parallelogram when $\overline{AD}$ and $\overline{BC}$ are used as bases.

For ▱*ABCD*, what is *DE* to the nearest tenth?



First, find the area of ▱*ABCD*. Then use the area formula a second time to find *DE*.

$A = bh$

$= 13(9) = 117$ Use base *AD* and height *CF*.

The area of ▱*ABCD* is 117 in.^{2}.

$A = bh$

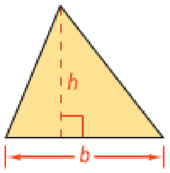
$117 = 9.4(DE)$ Use base *AB* and height *DE*.

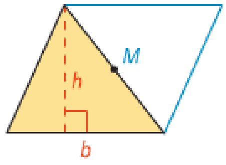
$DE = \frac{117}{9.4} \approx 12.4$

DE is about 12.4 in.

Got It? **2.** A parallelogram has sides 15 cm and 18 cm. The height corresponding to a 15-cm base is 9 cm. What is the height corresponding to an 18-cm base?

You can rotate a triangle about the midpoint of a side to form a parallelogram.





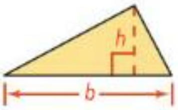
The area of the triangle is half the area of the parallelogram.

take note

Theorem 10-3 Area of a Triangle

The area of a triangle is half the product of a base and the corresponding height.

$$A = \frac{1}{2}bh$$



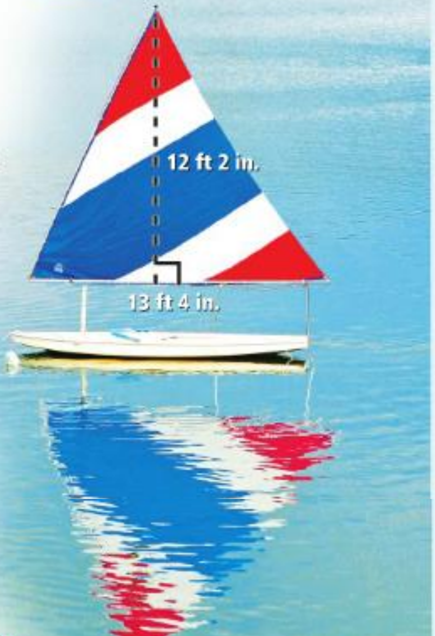
A **base of a triangle** can be any of its sides. The corresponding **height** is the length of the altitude to the line containing that base.

Problem 3 Finding the Area of a Triangle

Plan

Why do you need to convert the base and the height into inches?
You must convert them both because you can only multiply measurements with like units.

Sailing You want to make a triangular sail like the one at the right. How many square feet of material do you need?



Step 1 Convert the dimensions of the sail to inches.

$(12 \text{ ft} \cdot \frac{12 \text{ in.}}{1 \text{ ft}}) + 2 \text{ in.} = 146 \text{ in.}$ Use a conversion factor.

$(13 \text{ ft} \cdot \frac{12 \text{ in.}}{1 \text{ ft}}) + 4 \text{ in.} = 160 \text{ in.}$

Step 2 Find the area of the triangle.

$A = \frac{1}{2}bh$

$= \frac{1}{2}(160)(146)$ Substitute 160 for b and 146 for h.

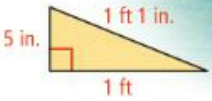
$= 11{,}680$ Simplify.

Step 3 Convert 11,680 in.^{2} to square feet.

$11{,}680 \text{ in.}^2 \cdot \frac{1 \text{ ft}}{12 \text{ in.}} \cdot \frac{1 \text{ ft}}{12 \text{ in.}} = 81\frac{1}{9} \text{ ft}^2$

You need $81\frac{1}{9}$ ft^2 of material.

Got It? **3.** What is the area of the triangle?

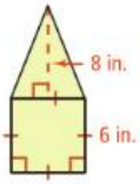


Problem 4 Finding the Area of an Irregular Figure

Plan

How do you know the length of the base of the triangle?
The lower part of the figure is a square. The base length of the triangle is the same as the base length of the square.

What is the area of the figure at the right?



Find the area of each part of the figure.

triangle area $= \frac{1}{2}bh = \frac{1}{2}(6)8 = 24$ in.^{2}

square area $= bh = 6(6) = 36$ in.^{2}

area of the figure $= 24$ in.^{2} $+ 36$ in.^{2} $= 60$ in.^{2}

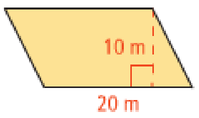
Got It? **4.** **Reasoning** Suppose the base lengths of the square and triangle in the figure above are doubled to 12 in., but the height of each polygon remains the same. How is the area of the figure affected?

Lesson Check

Do you know HOW?

Find the area of each parallelogram.

1.



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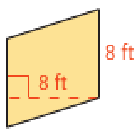
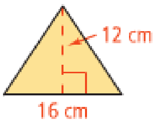
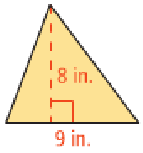


Find the area of each triangle.

3.



4.

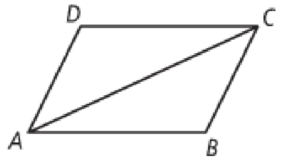


Do you UNDERSTAND?

5. Vocabulary Does an altitude of a triangle have to lie inside the triangle? Explain.

6. Writing How can you show that a parallelogram and a rectangle with the same bases and heights have equal areas?

7. $\square ABCD$ is divided into two triangles along diagonal $\overline{AC}$. If you know the area of the parallelogram, how do you find the area of $\triangle ABC$?



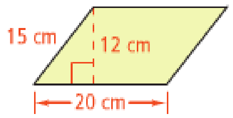
Practice and Problem-Solving Exercises

A Practice

Find the area of each parallelogram.

See Problem 1.

8.



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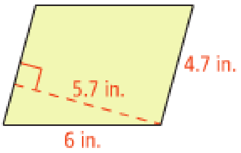

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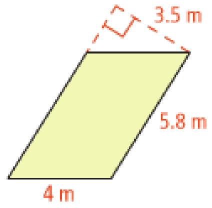
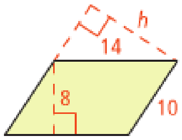


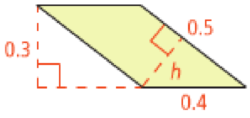
Find the value of *h* for each parallelogram.

See Problem 2.

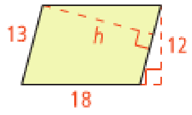
11.



12.



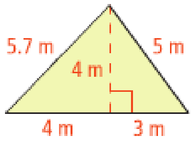
13.



Find the area of each triangle.

See Problem 3.

14.



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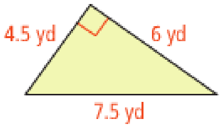

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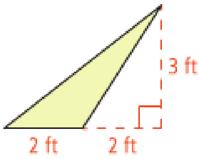


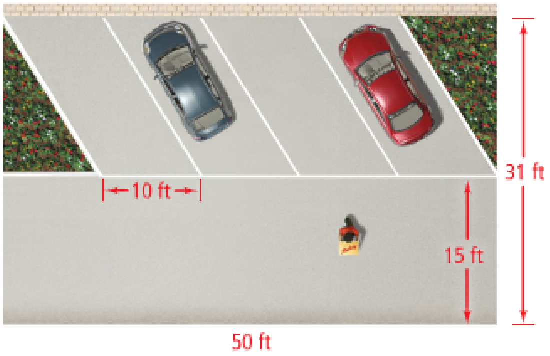
See Problem 4.

17. **Urban Design** A bakery has a 50 ft-by-31 ft parking lot. The four parking spaces are congruent parallelograms, the driving region is a rectangle, and the two areas for flowers are congruent triangles.

a. Find the area of the paved surface by adding the areas of the driving region and the four parking spaces.

b. Describe another method for finding the area of the paved surface.

c. Use your method from part (b) to find the area. Then compare answers from parts (a) and (b) to check your work.

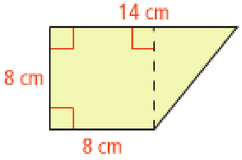


B Apply

18. The area of a parallelogram is 24 in.^{2} and the height is 6 in. Find the length of the corresponding base.

19. What is the area of the figure at the right?

(A) $64\ \text{cm}^2$ (B) $88\ \text{cm}^2$ (C) $96\ \text{cm}^2$ (D) $112\ \text{cm}^2$

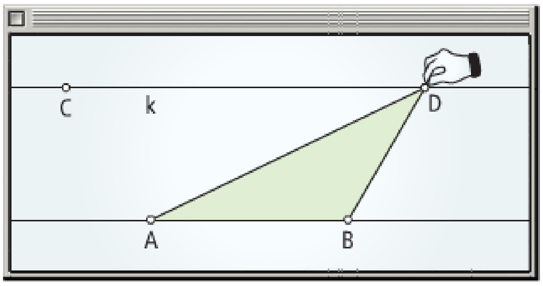


20. A right isosceles triangle has area 98 cm^2. Find the length of each leg.

21. **Algebra** The area of a triangle is 108 in.^{2}. A base and corresponding height are in the ratio 3 : 2. Find the length of the base and the corresponding height.

22. **Think About a Plan** Ki used geometry software to create the figure at the right. She constructed $\overleftrightarrow{AB}$ and a point C not on $\overleftrightarrow{AB}$. Then she constructed line k parallel to $\overleftrightarrow{AB}$ through point C. Next, Ki constructed point D on line k as well as $\overline{AD}$ and $\overline{BD}$. She dragged point D along line k to manipulate $\triangle ABD$. How does the area of $\triangle ABD$ change? Explain.

- Which dimensions of the triangle change when Ki drags point D?
- Do the lengths of AD and BD matter when calculating area?



23. **Open-Ended** Using graph paper, draw an acute triangle, an obtuse triangle, and a right triangle, each with area 12 units^2.

Find the area of each figure.

24. ▱$ABJF$

25. $\triangle BDJ$

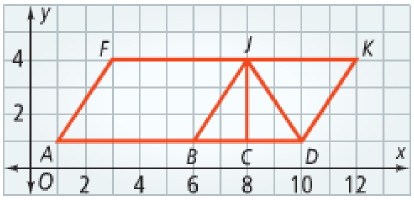
26. $\triangle DKJ$

27. ▱$BDKJ$

28. ▱$ADKF$

29. $\triangle BCJ$

30. trapezoid $ADJF$



31. **Reasoning** Suppose the height of a triangle is tripled. How does this affect the area of the triangle? Explain.

For Exercises 32–35, (a) graph the lines and (b) find the area of the triangle enclosed by the lines.

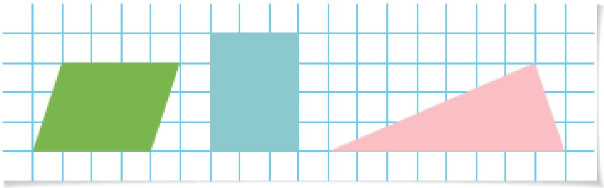
32. $y = x, x = 0, y = 7$

33. $y = x + 2, y = 2, x = 6$

34. $y = -\frac{1}{2}x + 3, y = 0, x = -2$

35. $y = \frac{3}{4}x - 2, y = -2, x = 4$

36. **Probability** Your friend drew these three figures on a grid. A fly lands at random at a point on the grid.



a. **Writing** Is the fly more likely to land on one of the figures or on the blank grid? Explain.

b. Suppose you know the fly lands on one of the figures. Is the fly more likely to land on one figure than on another? Explain.

Coordinate Geometry **Find the area of a polygon with the given vertices.**

37. $A(3, 9), B(8, 9), C(2, -3), D(-3, -3)$

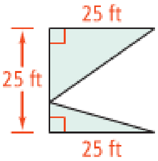
38. $E(1, 1), F(4, 5), G(11, 5), H(8, 1)$

39. $D(0, 0), E(2, 4), F(6, 4), G(6, 0)$

40. $K(-7, -2), L(-7, 6), M(1, 6), N(7, -2)$

Find the area of each figure.

41.



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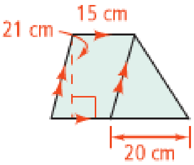

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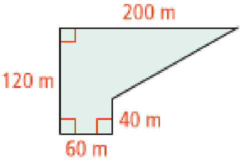


Challenge

History **The Greek mathematician Heron is most famous for this formula for the area of a triangle in terms of the lengths of its sides *a*, *b*, and *c*.**

$$A = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \tfrac{1}{2}(a + b + c)$$

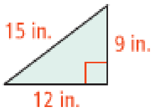
Use Heron's Formula and a calculator to find the area of each triangle. Round your answer to the nearest whole number.

44. $a = 8$ in., $b = 9$ in., $c = 10$ in.

45. $a = 15$ m, $b = 17$ m, $c = 21$ m

46. **a.** Use Heron's Formula to find the area of this triangle.

b. Verify your answer to part (a) by using the formula $A = \frac{1}{2}bh$.



Standardized Test Prep

SAT/ACT

47. The lengths of the sides of a right triangle are 10 in., 24 in., and 26 in. What is the area of the triangle?

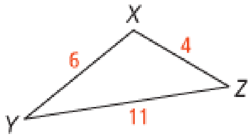
Ⓐ 116 in.^{2} Ⓑ 120 in.^{2} Ⓒ 130 in.^{2} Ⓓ 156 in.^{2}

48. In quadrilateral *ABCD*, $AB \cong BC \cong CD \cong DA$. Which type of quadrilateral could *ABCD* never be classified as?

Ⓕ square Ⓖ rectangle Ⓗ rhombus Ⓘ kite

Short Response

49. Are the side lengths of $\triangle XYZ$ possible? Explain.

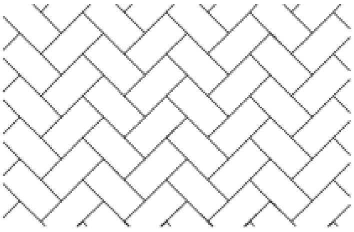


Mixed Review

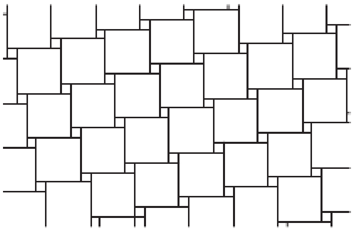
List the symmetries in each tessellation.

See Lesson 9-7.

50.

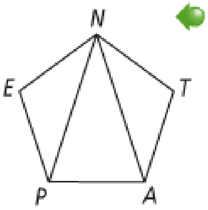


51.



The base of the isosceles triangle is also a side of a regular pentagon *PENTA*. Find the measure of each angle.

See Lesson 4-5.



52. $\angle APE$

53. $\angle APN$

54. $\angle PAN$

55. $\angle PNA$

56. $\angle EPN$

57. $\angle ANT$

Get Ready! **To prepare for Lesson 10-2, do Exercises 58–62.**

Write the formula for the area of each type of figure.

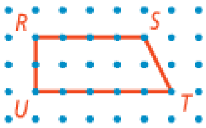
See Lesson 10-1.

58. a rectangle

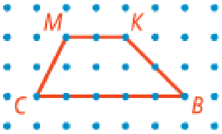
59. a triangle

Find the area of each trapezoid by using the formulas for area of a rectangle and area of a triangle.

60.



61.



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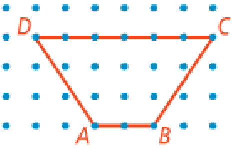


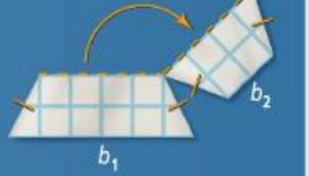
10-2 Areas of Trapezoids, Rhombuses, and Kites

Objective To find the area of a trapezoid, rhombus, or kite

Rearranging figures into familiar shapes is an example of the Solve a Simpler Problem strategy.

SOLVE IT! **Getting Ready!**

Draw a trapezoid on a sheet of graph paper. Label the bases b_1 and b_2. Draw its midsegment. Cut out the trapezoid, and then cut it along the midsegment. Rotate the top part of the trapezoid 180° so that b_1 and b_2 now form one long base. How can you use this new figure to find the area of the trapezoid? Explain your reasoning.



Lesson Vocabulary
- height of a trapezoid

Essential Understanding You can find the area of a trapezoid when you know its height and the lengths of its bases.

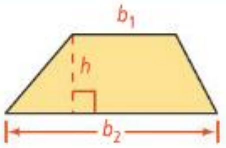
The **height of a trapezoid** is the perpendicular distance between the bases.

take note

Theorem 10-4 Area of a Trapezoid

The area of a trapezoid is half the product of the height and the sum of the bases.

$$A = \frac{1}{2}h(b_1 + b_2)$$



Problem 1 Area of a Trapezoid

Plan

Which borders of Nevada can you use as the bases of a trapezoid?
The two parallel sides of Nevada form the bases of a trapezoid.

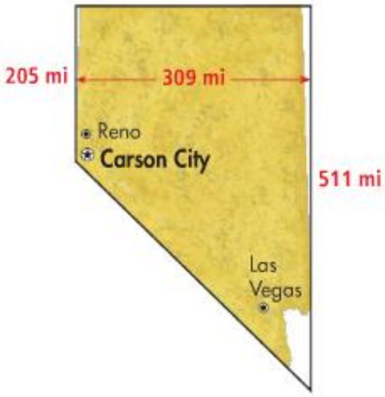
Geography **What is the approximate area of Nevada?**

$A = \frac{1}{2}h(b_1 + b_2)$ Use the formula for area of a trapezoid.

$= \frac{1}{2}(309)(205 + 511)$ Substitute 309 for h, 205 for b_1, and 511 for b_2.

$= 110{,}622$ Simplify.

The area of Nevada is about 110,600 mi^2.



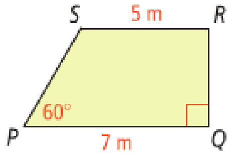
Got It? **1.** What is the area of a trapezoid with height 7 cm and bases 12 cm and 15 cm?

Problem 2 Finding Area Using a Right Triangle

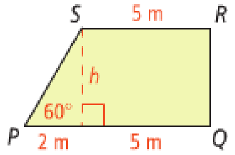
Think

How are the sides related in a 30°-60°-90° triangle?
The length of the hypotenuse is 2 times the length of the shorter leg, and the longer leg is $\sqrt{3}$ times the length of the shorter leg.

What is the area of trapezoid *PQRS*?



You can draw an altitude that divides the trapezoid into a rectangle and a 30°-60°-90° triangle. Since the opposite sides of a rectangle are congruent, the longer base of the trapezoid is divided into segments of lengths 2 m and 5 m.



$h = 2\sqrt{3}$ longer leg = shorter leg · $\sqrt{3}$

$A = \frac{1}{2}h(b_1 + b_2)$ Use the trapezoid area formula.

$= \frac{1}{2}(2\sqrt{3})(7 + 5)$ Substitute $2\sqrt{3}$ for h, 7 for b_1, and 5 for b_2.

$= 12\sqrt{3}$ Simplify.

The area of trapezoid *PQRS* is $12\sqrt{3}$ m^2.

Got It? 2. Reasoning In Problem 2, suppose h decreases so that $m\angle P = 45$ while angles R and Q and the bases stay the same. What is the area of trapezoid *PQRS*?

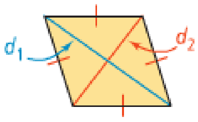
Essential Understanding You can find the area of a rhombus or a kite when you know the lengths of its diagonals.

take note

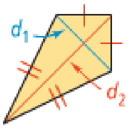
Theorem 10-5 Area of a Rhombus or a Kite

The area of a rhombus or a kite is half the product of the lengths of its diagonals.

$$A = \frac{1}{2}d_1d_2$$



Rhombus



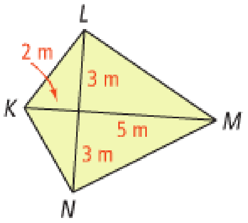
Kite

Problem 3 Finding the Area of a Kite

Think

Do you need to know the side lengths of the kite to find its area?
No. You only need the lengths of the diagonals.

What is the area of kite *KLMN*?



Find the lengths of the two diagonals:
$KM = 2 + 5 = 7$ m and $LN = 3 + 3 = 6$ m.

$A = \frac{1}{2}d_1d_2$ Use the formula for area of a kite.

$= \frac{1}{2}(7)(6)$ Substitute 7 for d_1 and 6 for d_2.

$= 21$ Simplify.

The area of kite *KLMN* is 21 m^2.

Got It? 3. What is the area of a kite with diagonals that are 12 in. and 9 in. long?

Problem 4 Finding the Area of a Rhombus

Car Pooling The High Occupancy Vehicle (HOV) lane is marked by a series of "diamonds," or rhombuses painted on the pavement. What is the area of the HOV lane diamond shown at the right?

Think

How can you find the length of $\overline{AB}$?

$\overline{AB}$ is a leg of right $\triangle ABC$. You can use the Pythagorean Theorem, $a^2 + b^2 = c^2$, to find its length.

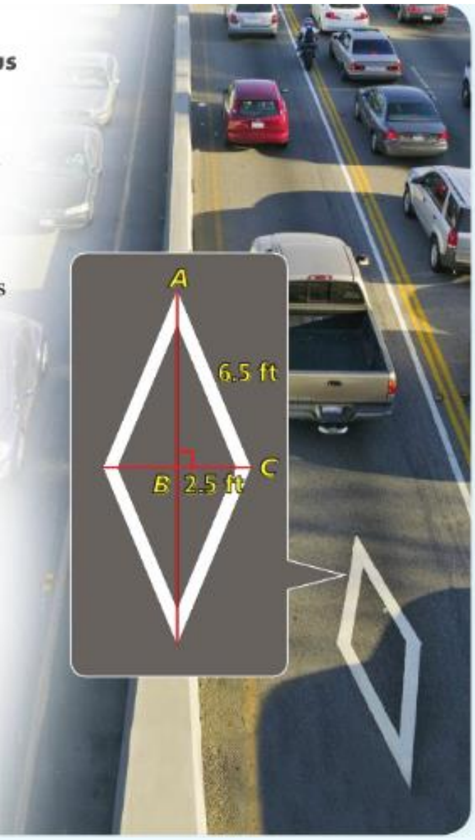
$\triangle ABC$ is a right triangle. Using the Pythagorean Theorem, $AB = \sqrt{6.5^2 - 2.5^2} = 6$. Since the diagonals of a rhombus bisect each other, the diagonals of the HOV lane diamond are 5 ft and 12 ft.

$A = \frac{1}{2}d_1d_2$ Use the formula for area of a rhombus.

$= \frac{1}{2}(5)(12)$ Substitute 5 for d_1 and 12 for d_2.

$= 30$ Simplify.

The area of the HOV lane diamond is 30 ft^2.



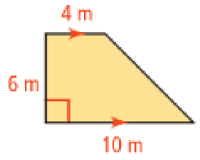
Got It? 4. A rhombus has sides 10 cm long. If the longer diagonal is 16 cm, what is the area of the rhombus?

Lesson Check

Do you know HOW?

Find the area of each figure.

1.



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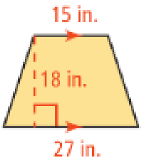

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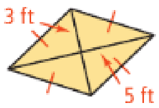

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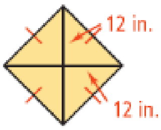

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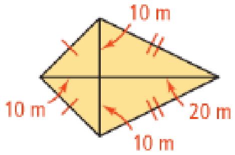

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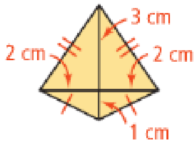
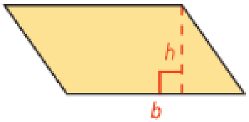
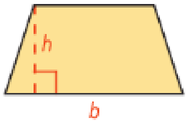


Do you UNDERSTAND?

7. **Vocabulary** Can a trapezoid and a parallelogram with the same base and height have the same area? Explain.





8. **Reasoning** Do you need to know all the side lengths to find the area of a trapezoid?

9. **Reasoning** Can you find the area of a rhombus if you only know the lengths of its sides? Explain.

10. **Reasoning** Do you need to know the lengths of the sides to find the area of a kite? Explain.

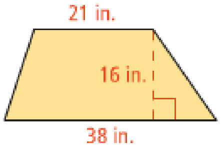
Practice and Problem-Solving Exercises

A Practice

Find the area of each trapezoid

See Problem 1.

11.



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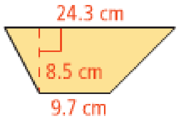

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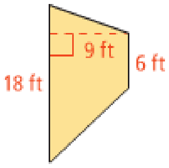


14. Find the area of a trapezoid with bases 12 cm and 18 cm and height 10 cm.

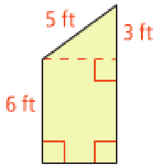
15. Find the area of a trapezoid with bases 2 ft and 3 ft and height $\frac{1}{3}$ ft.

16. **Geography** The border of Tennessee resembles a trapezoid with bases 340 mi and 440 mi and height 110 mi. Estimate the area of Tennessee by finding the area of the trapezoid.

Find the area of each trapezoid. If your answer is not an integer, leave it in simplest radical form.

See Problem 2.

17.



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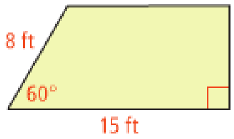

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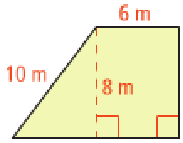
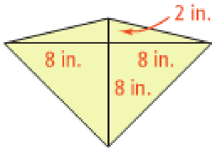


Find the area of each kite.

See Problem 3.

20.



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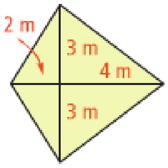

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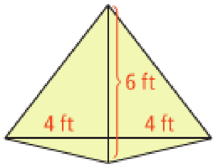
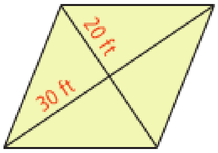


Find the area of each rhombus.

See Problem 4.

23.



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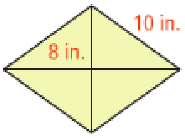

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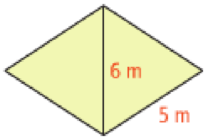
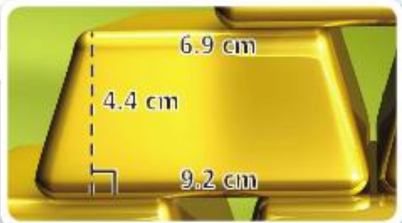


B Apply

26. **Think About a Plan** A trapezoid has two right angles, 12-m and 18-m bases, and an 8-m height. Sketch the trapezoid and find its perimeter and area.

- Are the right angles consecutive or opposite angles?
- How does knowing the height help you find the perimeter?

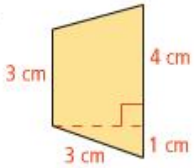
27. **Metallurgy** The end of a gold bar has the shape of a trapezoid with the measurements shown. Find the area of the end.



28. **Open-Ended** Draw a kite. Measure the lengths of its diagonals. Find its area.

Find the area of each trapezoid to the nearest tenth.

29.



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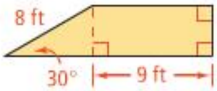

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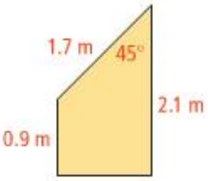
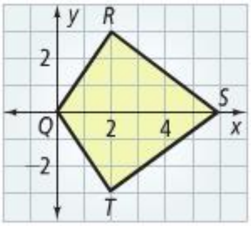


Coordinate Geometry **Find the area of quadrilateral *QRST*.**

32.



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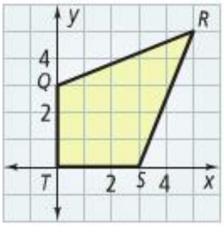

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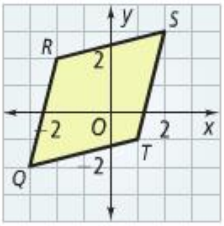
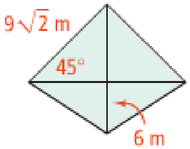


35. What is the area of the kite at the right?



Ⓐ 90 m^2

Ⓑ 108 m^2

Ⓒ 135 m^2

Ⓓ 216 m^2

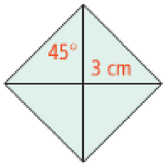
36. **a.** **Coordinate Geometry** Graph the lines $x = 0$, $x = 6$, $y = 0$, and $y = x + 4$.

b. What type of quadrilateral do the lines form?

c. Find the area of the quadrilateral.

Find the area of each rhombus. Leave your answer in simplest radical form.

37.



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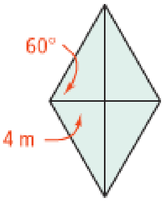

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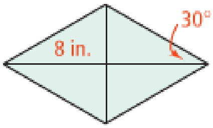
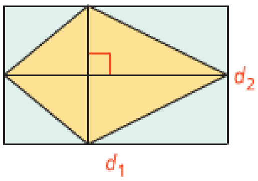


40. **Visualization** The kite has diagonals d_1 and d_2 congruent to the sides of the rectangle. Explain why the area of the kite is $\frac{1}{2}d_1d_2$.



41. Draw a trapezoid. Label its bases b_1 and b_2 and its height h. Then draw a diagonal of the trapezoid.

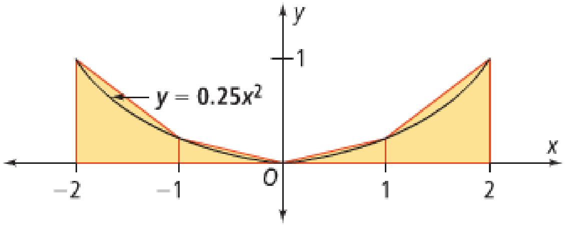
a. Write equations for the area of each of the two triangles formed.

b. **Writing** Explain how you can justify the trapezoid area formula using the areas of the two triangles.

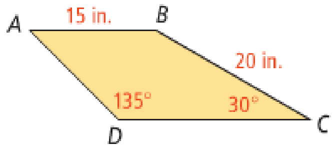
Challenge

42. Algebra One base of a trapezoid is twice the other. The height is the average of the two bases. The area is 324 cm^2. Find the height and the bases. (*Hint:* Let the smaller base be x.)

43. Sports Ty wants to paint one side of the skateboarding ramp he built. The ramp is 4 m wide. Its surface is modeled by the equation $y = 0.25x^2$. Use the trapezoids and triangles shown to estimate the area to be painted.



44. In trapezoid $ABCD$ at the right, $\overline{AB} \parallel \overline{DC}$. Find the area of $ABCD$.



Standardized Test Prep

SAT/ACT

45. The area of a kite is 120 cm^2. The length of one diagonal is 20 cm. What is the length of the other diagonal?

Ⓐ 12 cm Ⓑ 20 cm Ⓒ 24 cm Ⓓ 48 cm

46. $\triangle ABC \sim \triangle XYZ$. $AB = 6$, $BC = 3$, and $CA = 7$. Which of the following are NOT possible dimensions of $\triangle XYZ$?

Ⓕ $XY = 3$, $YZ = 1.5$, $ZX = 3.5$

Ⓗ $XY = 10$, $YZ = 7$, $ZX = 11$

Ⓖ $XY = 9$, $YZ = 4.5$, $ZX = 10.5$

Ⓘ $XY = 18$, $YZ = 9$, $ZX = 21$

Short Response

47. Draw an angle. Construct a congruent angle and its bisector.

Mixed Review

48. Find the area of a right isosceles triangle that has one leg of length 12 cm. **See Lesson 10-1.**

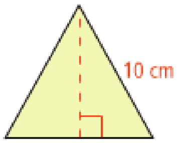
49. A right isosceles triangle has area 112.5 ft^2. Find the length of each leg.

50. Find the measure of an interior angle of a regular nonagon. **See Lesson 6-1.**

Get Ready! **To prepare for Lesson 10-3, do Exercises 51–53.**

Find the area of each regular polygon. Leave radicals in simplest form. **See Lesson 8-2.**

51.



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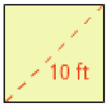

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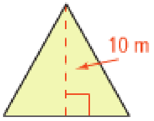


10-3 Areas of Regular Polygons

Objective To find the area of a regular polygon

Let's see. What figure encloses the greatest area for a given perimeter?

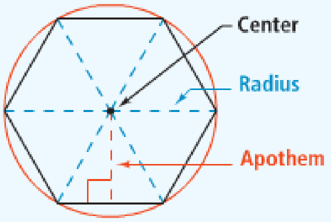
SOLVE IT! Getting Ready!

You want to build a koi pond. For the border, you plan to use 3-ft-long pieces of wood. You have 12 pieces that you can connect together at any angle, including a straight angle. If you want to maximize the area of the pond, in what shape should you arrange the pieces? Explain your reasoning.

Lesson Vocabulary
- radius of a regular polygon
- apothem

The Solve It involves the area of a polygon.

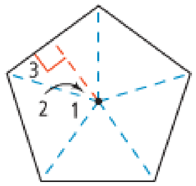
Essential Understanding The area of a regular polygon is related to the distance from the center to a side.



You can circumscribe a circle about any regular polygon. The center of a regular polygon is the center of the circumscribed circle. The **radius of a regular polygon** is the distance from the center to a vertex. The **apothem** is the perpendicular distance from the center to a side.

Problem 1 Finding Angle Measures

The figure at the right is a regular pentagon with radii and an apothem drawn. What is the measure of each numbered angle?



$m\angle 1 = \frac{360}{5} = 72$ Divide 360 by the number of sides.

$m\angle 2 = \frac{1}{2}m\angle 1$ The apothem bisects the vertex angle of the isosceles triangle formed by the radii.

$= \frac{1}{2}(72) = 36$

$90 + 36 + m\angle 3 = 180$ The sum of the measures of the angles of a triangle is 180.

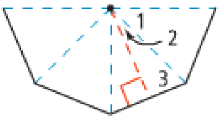
$m\angle 3 = 54$

$m\angle 1 = 72$, $m\angle 2 = 36$, and $m\angle 3 = 54$.

Think

How do you know the radii make isosceles triangles? Since the pentagon is a regular polygon, the radii are congruent. So, the triangle made by two adjacent radii and a side of the polygon is an isosceles triangle.

Got It? **1.** At the right, a portion of a regular octagon has radii and an apothem drawn. What is the measure of each numbered angle?

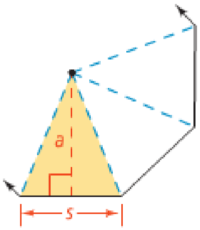


take note

Postulate 10-1

If two figures are congruent, then their areas are equal.

Suppose you have a regular n-gon with side s. The radii divide the figure into n congruent isosceles triangles. By Postulate 10-1, the areas of the isosceles triangles are equal. Each triangle has a height of a and a base of length s, so the area of each triangle is $\frac{1}{2}as$.



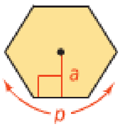
Since there are n congruent triangles, the area of the n-gon is $A = n \cdot \frac{1}{2}as$. The perimeter p of the n-gon is the number of sides n times the length of a side s, or ns. By substitution, the area can be expressed as $A = \frac{1}{2}ap$.

take note

Theorem 10-6 Area of a Regular Polygon

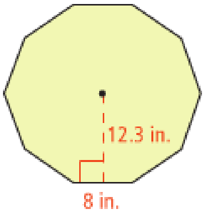
The area of a regular polygon is half the product of the apothem and the perimeter.

$$A = \frac{1}{2}ap$$



Problem 2 Finding the Area of a Regular Polygon

What is the area of the regular decagon at the right?



Plan

What do you know about the regular decagon?
A decagon has 10 sides, so $n = 10$. From the diagram, you know that the apothem a is 12.3 in., and the side length s is 8 in.

Step 1 Find the perimeter of the regular decagon.

$p = ns$ Use the formula for the perimeter of an n-gon.

$= 10(8)$ Substitute 10 for n and 8 for s.

$= 80$ in.

Step 2 Find the area of the regular decagon.

$A = \frac{1}{2}ap$ Use the formula for the area of a regular polygon.

$= \frac{1}{2}(12.3)(80)$ Substitute 12.3 for a and 80 for p.

$= 492$

The regular decagon has an area of 492 in.^{2}.

Got It? **2.** **a.** What is the area of a regular pentagon with an 8-cm apothem and 11.6-cm sides?

b. **Reasoning** If the side of a regular polygon is reduced to half its length, how does the perimeter of the polygon change? Explain.

Problem 3 Using Special Triangles to Find Area

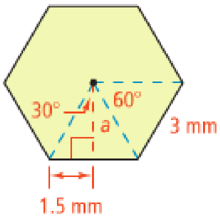
Zoology **A honeycomb is made up of regular hexagonal cells. The length of a side of a cell is 3 mm. What is the area of a cell?**

Know
You know the length of a side, which you can use to find the perimeter.

Need
The apothem

Plan
Draw a diagram to help find the apothem. Then use the area formula for a regular polygon.

Step 1 Find the apothem.



The radii form six 60° angles at the center, so you can use a 30°-60°-90° triangle to find the apothem.

$a = 1.5\sqrt{3}$ longer leg $= \sqrt{3} \cdot$ shorter leg

Step 2 Find the perimeter.

$p = ns$ Use the formula for the perimeter of an n-gon.

$= 6(3)$ Substitute 6 for n and 3 for s.

$= 18$ mm

Step 3 Find the area.

$A = \frac{1}{2}ap$ Use the formula for the area of a regular polygon.

$= \frac{1}{2}(1.5\sqrt{3})(18)$ Substitute $1.5\sqrt{3}$ for a and 18 for p.

≈ 23.3826859 Use a calculator.

The area is about 23 mm^2.

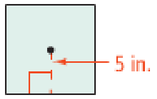
Got It? **3.** The side of a regular hexagon is 16 ft. What is the area of the hexagon? Round your answer to the nearest square foot.

Lesson Check

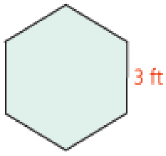
Do you know HOW?

What is the area of each regular polygon? Round your answer to the nearest tenth.

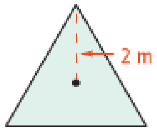
1.



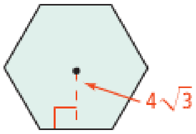
2.



3.



4.



Do you UNDERSTAND?

5. **Vocabulary** What is the difference between a radius and an apothem?

6. What is the relationship between the side length and the apothem in each figure?
  a. a square
  b. a regular hexagon
  c. an equilateral triangle

7. **Error Analysis** Your friend says you can use special triangles to find the apothem of any regular polygon. What is your friend's error? Explain.

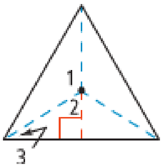
Practice and Problem-Solving Exercises

A Practice

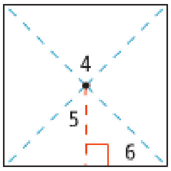
Each regular polygon has radii and apothem as shown. Find the measure of each numbered angle.

See Problem 1.

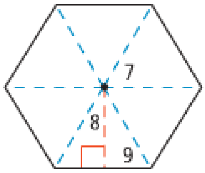
8.



9.



10.



Find the area of each regular polygon with the given apothem *a* and side length *s*.

See Problem 2.

11. pentagon, $a = 24.3$ cm, $s = 35.3$ cm

12. 7-gon, $a = 29.1$ ft, $s = 28$ ft

13. octagon, $a = 60.4$ in., $s = 50$ in.

14. nonagon, $a = 27.5$ in., $s = 20$ in.

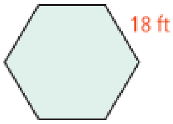
15. decagon, $a = 19$ m, $s = 12.3$ m

16. dodecagon, $a = 26.1$ cm, $s = 14$ cm

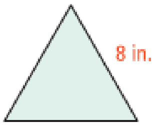
Find the area of each regular polygon. Round your answer to the nearest tenth.

See Problem 3.

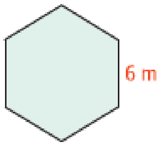
17.



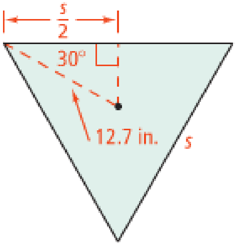
18.



19.

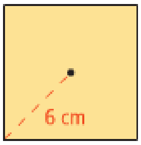


20. **Art** You are painting a mural of colored equilateral triangles. The radius of each triangle is 12.7 in. What is the area of each triangle to the nearest square inch?



Find the area of each regular polygon with the given radius or apothem. If your answer is not an integer, leave it in simplest radical form.

21.



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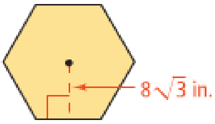
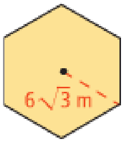


23.



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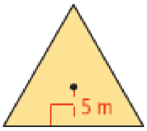
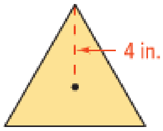


25.



B Apply

Find the measures of the angles formed by (a) two consecutive radii and (b) a radius and a side of the given regular polygon.

26. pentagon

27. octagon

28. nonagon

29. dodecagon

30. **Satellites** One of the smallest space satellites ever developed has the shape of a pyramid. Each of the four faces of the pyramid is an equilateral triangle with sides about 13 cm long. What is the area of one equilateral triangular face of the satellite? Round your answer to the nearest whole number.

31. **Think About a Plan** The gazebo in the photo is built in the shape of a regular octagon. Each side is 8 ft long, and the enclosed area is 310.4 ft^2. What is the length of the apothem?

- How can you *draw a diagram* to help you solve the problem?
- How can you use the area of a regular polygon formula?

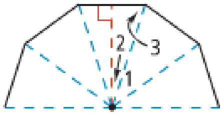
32. A regular hexagon has perimeter 120 m. Find its area.

33. The area of a regular polygon is 36 in.^2. Find the length of a side if the polygon has the given number of sides. Round your answer to the nearest tenth.

a. 3 **b.** 4 **c.** 6

d. **Estimation** Suppose the polygon is a pentagon. What would you expect the length of a side to be? Explain.

34. A portion of a regular decagon has radii and an apothem drawn. Find the measure of each numbered angle.



35. **Writing** Explain why the radius of a regular polygon is greater than the apothem.

36. **Constructions** Use a compass to construct a circle.

a. Construct two perpendicular diameters of the circle.

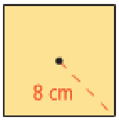
b. Construct diameters that bisect each of the four right angles.

c. Connect the consecutive points where the diameters intersect the circle. What regular polygon have you constructed?

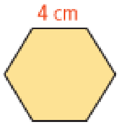
d. **Reasoning** How can a circle help you construct a regular hexagon?

Find the area of each regular polygon. Show your answers in simplest radical form and rounded to the nearest tenth.

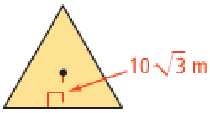
37.



38.



39.



40. To find the area of an equilateral triangle, you can use the formula $A = \frac{1}{2}bh$ or $A = \frac{1}{2}ap$. A third way to find the area of an equilateral triangle is to use the formula $A = \frac{1}{4}s^2\sqrt{3}$. Verify the formula $A = \frac{1}{4}s^2\sqrt{3}$ in two ways as follows:

a. Find the area of Figure 1 using the formula $A = \frac{1}{2}bh$.

b. Find the area of Figure 2 using the formula $A = \frac{1}{2}ap$.

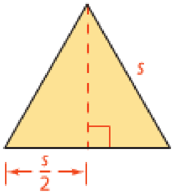


Figure 1

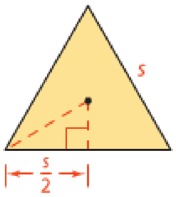


Figure 2

41. For Problem 1 on page 629, write a proof that the apothem bisects the vertex angle of an isosceles triangle formed by two radii.

Proof

Challenge

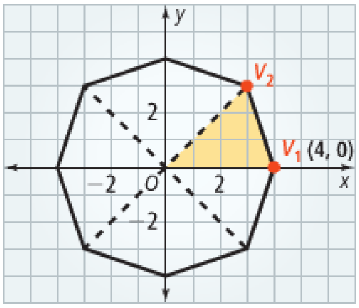
42. **Proof** Prove that the bisectors of the angles of a regular polygon are concurrent and that they are, in fact, radii of the polygon. (*Hint:* For regular n-gon $ABCDE\ldots$, let P be the intersection of the bisectors of $\angle ABC$ and $\angle BCD$. Show that $\overline{DP}$ must be the bisector of $\angle CDE$.)

43. **Coordinate Geometry** A regular octagon with center at the origin and radius 4 is graphed in the coordinate plane.

a. Since V_2 lies on the line $y = x$, its x- and y-coordinates are equal. Use the Distance Formula to find the coordinates of V_2 to the nearest tenth.

b. Use the coordinates of V_2 and the formula $A = \frac{1}{2}bh$ to find the area of $\triangle V_1OV_2$ to the nearest tenth.

c. Use your answer to part (b) to find the area of the octagon to the nearest whole number.



Standardized Test Prep

SAT/ACT

44. What is the area of a regular pentagon with an apothem of 25.1 mm and perimeter of 182 mm?

(A) 913.6 mm^2 (B) 2284.1 mm^2 (C) 3654.6 mm^2 (D) 4568.2 mm^2

45. What is the most precise name for a regular polygon with four right angles?

(F) square (G) parallelogram (H) trapezoid (I) rectangle

46. $\triangle ABC$ has coordinates $A(-2, 4)$, $B(3, 1)$, and $C(0, -2)$. If you reflect $\triangle ABC$ across the x-axis, what are the coordinates of the vertices of the image $\triangle A'B'C'$?

(A) $A'(2, 4)$, $B'(-3, 1)$, $C'(0, -2)$ (C) $A'(4, -2)$, $B'(1, 3)$, $C'(-2, 0)$

(B) $A'(-2, -4)$, $B'(3, -1)$, $C'(0, 2)$ (D) $A'(4, 2)$, $B'(1, -3)$, $C'(-2, 0)$

Short Response

47. An equilateral triangle on a coordinate grid has vertices at (0, 0) and (4, 0). What are the possible locations of the third vertex?

Mixed Review

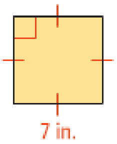
48. What is the area of a kite with diagonals 8 m and 11.5 m? **See Lesson 10-2.**

49. The area of a trapezoid is 42 m^2. The trapezoid has a height of 7 m and one base of 4 m. What is the length of the other base?

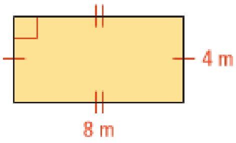
Get Ready! **To prepare for Lesson 10-4, do Exercises 50–52.** **See Lesson 1-8.**

Find the perimeter and area of each figure.

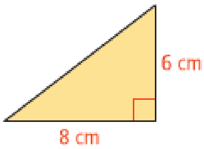
50.



51.

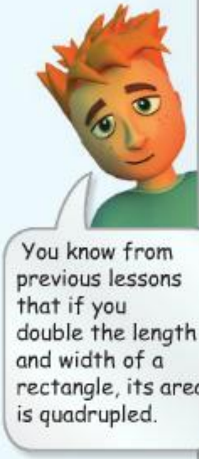


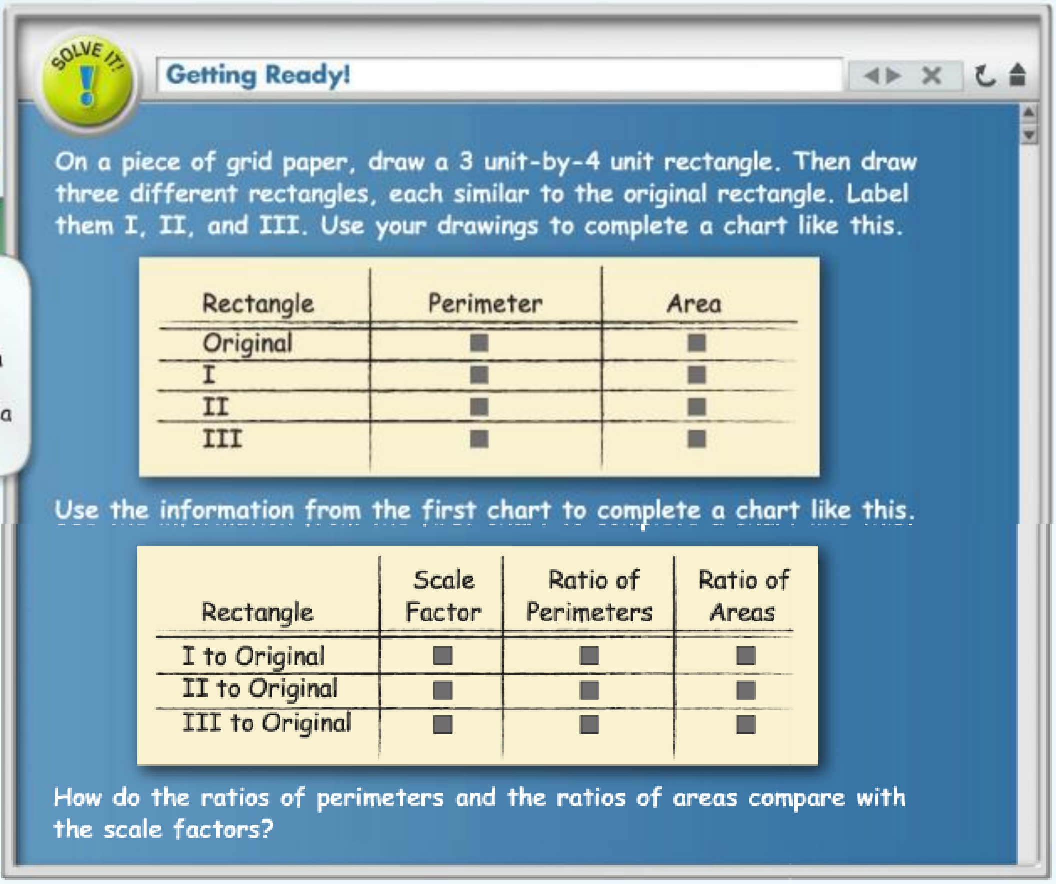
52.



10-4 Perimeters and Areas of Similar Figures

Objective To find the perimeters and areas of similar polygons


You know from previous lessons that if you double the length and width of a rectangle, its area is quadrupled.



On a piece of grid paper, draw a 3 unit-by-4 unit rectangle. Then draw three different rectangles, each similar to the original rectangle. Label them I, II, and III. Use your drawings to complete a chart like this.

Rectangle	Perimeter	Area
Original	■	■
I	■	■
II	■	■
III	■	■

Use the information from the first chart to complete a chart like this.

Rectangle	Scale Factor	Ratio of Perimeters	Ratio of Areas
I to Original	■	■	■
II to Original	■	■	■
III to Original	■	■	■

How do the ratios of perimeters and the ratios of areas compare with the scale factors?

Dynamic Activity
Perimeters and Areas of Similar Figures

In the Solve It, you compared the areas of similar figures.

Essential Understanding You can use ratios to compare the perimeters and areas of similar figures.

take note

Theorem 10-7 Perimeters and Areas of Similar Figures

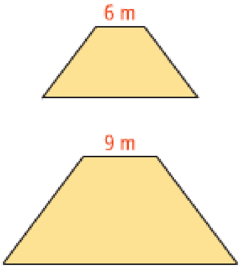
If the scale factor of two similar figures is $\frac{a}{b}$, then

(1) the ratio of their perimeters is $\frac{a}{b}$ and

(2) the ratio of their areas is $\frac{a^2}{b^2}$.

Problem 1 Finding Ratios in Similar Figures

The trapezoids at the right are similar. The ratio of the lengths of corresponding sides is $\frac{6}{9}$, or $\frac{2}{3}$.



Plan

How do you find the scale factor?
Write the ratio of the lengths of two corresponding sides.

A What is the ratio (smaller to larger) of the perimeters?

The ratio of the perimeters is the same as the ratio of corresponding sides, which is $\frac{2}{3}$.

B What is the ratio (smaller to larger) of the areas?

The ratio of the areas is the square of the ratio of corresponding sides, which is $\frac{2^2}{3^2}$, or $\frac{4}{9}$.

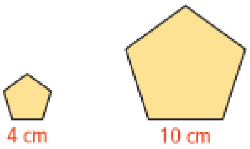
Got It? **1.** Two similar polygons have corresponding sides in the ratio 5 : 7.
 a. What is the ratio (larger to smaller) of their perimeters?
 b. What is the ratio (larger to smaller) of their areas?

When you know the area of one of two similar polygons, you can use a proportion to find the area of the other polygon.

Problem 2 Finding Areas Using Similar Figures

Multiple Choice **The area of the smaller regular pentagon is about 27.5 cm^2. What is the best approximation for the area of the larger regular pentagon?**

Ⓐ 11 cm^2 Ⓑ 69 cm^2 Ⓒ 172 cm^2 Ⓓ 275 cm^2



Think

Can you eliminate any answer choices immediately?
Yes. Since the area of the smaller pentagon is 27.5 cm^2, you know that the area of the larger pentagon must be greater than that, so you can eliminate choice A.

Regular pentagons are similar because all angles measure 108 and all sides in each pentagon are congruent. Here the ratio of corresponding side lengths is $\frac{4}{10}$, or $\frac{2}{5}$. The ratio of the areas is $\frac{2^2}{5^2}$, or $\frac{4}{25}$.

$\frac{4}{25} = \frac{27.5}{A}$ Write a proportion using the ratio of the areas.

$4A = 687.5$ Cross Products Property

$A = \frac{687.5}{4}$ Divide each side by 4.

$A = 171.875$ Simplify.

The area of the larger pentagon is about 172 cm^2. The correct answer is C.

Got It? **2.** The scale factor of two similar parallelograms is $\frac{3}{4}$. The area of the larger parallelogram is 96 in.^{2}. What is the area of the smaller parallelogram?

Problem 3 Applying Area Ratios

Think

Do you need to know the shapes of the two plots of land?
No. As long as the plots are similar, you can compare their areas using their scale factor.

Agriculture **During the summer, a group of high school students cultivated a plot of city land and harvested 13 bushels of vegetables that they donated to a food pantry. Next summer, the city will let them use a larger, similar plot of land. In the new plot, each dimension is 2.5 times the corresponding dimension of the original plot. How many bushels can the students expect to harvest next year?**

The ratio of the dimensions is 2.5 : 1. So, the ratio of the areas is $(2.5)^2 : 1^2$, or 6.25 : 1. With 6.25 times as much land next year, the students can expect to harvest 6.25(13), or about 81, bushels.

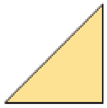
Got It? **3. a.** The scale factor of the dimensions of two similar pieces of window glass is 3 : 5. The smaller piece costs $2.50. How much should the larger piece cost?

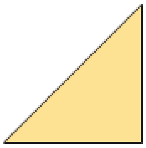
b. **Reasoning** In Problem 3, why is it important that *each* dimension is 2.5 times the corresponding dimension of the original plot? Explain.

When you know the ratio of the areas of two similar figures, you can work backward to find the ratio of their perimeters.

Problem 4 Finding Perimeter Ratios

The triangles at the right are similar. What is the scale factor? What is the ratio of their perimeters?


Area = 50 cm^2


Area = 98 cm^2

Know
The areas of the two similar triangles

Need
The scale factor

Plan
Write a proportion using the ratios of the areas.

$\frac{a^2}{b^2} = \frac{50}{98}$ Use $a^2 : b^2$ for the ratio of the areas.

$\frac{a^2}{b^2} = \frac{25}{49}$ Simplify.

$\frac{a}{b} = \frac{5}{7}$ Take the positive square root of each side.

The ratio of the perimeters equals the scale factor 5 : 7.

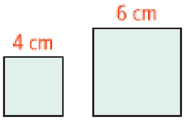
Got It? **4.** The areas of two similar rectangles are 1875 ft^2 and 135 ft^2. What is the ratio of their perimeters?

Lesson Check

Do you know HOW?

The figures in each pair are similar. What is the ratio of the perimeters and the ratio of the areas?

1.



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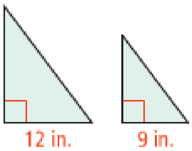


3. In Exercise 2, if the area of the smaller triangle is about 39 ft^2, what is the area of the larger triangle to the nearest tenth?

4. The areas of two similar rhombuses are 48 m^2 and 128 m^2. What is the ratio of their perimeters?

Do you UNDERSTAND?

5. **Reasoning** How does the ratio of the areas of two similar figures compare to the ratio of their perimeters? Explain.

6. **Reasoning** The area of one rectangle is twice the area of another. What is the ratio of their perimeters? How do you know?

7. **Error Analysis** Your friend says that since the ratio of the perimeters of two polygons is $\frac{1}{2}$, the area of the smaller polygon must be one half the area of the larger polygon. What is wrong with this statement? Explain.

8. **Compare and Contrast** How is the relationship between the areas of two congruent figures different from the relationship between the areas of two similar figures?

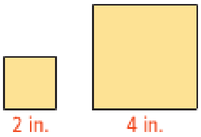
Practice and Problem-Solving Exercises

A Practice

The figures in each pair are similar. Compare the first figure to the second. Give the ratio of the perimeters and the ratio of the areas.

See Problem 1.

9.



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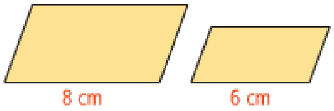

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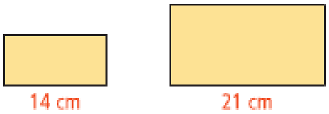

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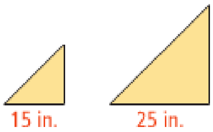
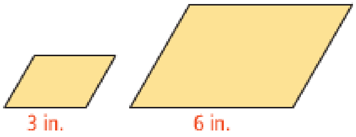


The figures in each pair are similar. The area of one figure is given. Find the area of the other figure to the nearest whole number.

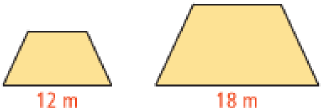
See Problem 2.

13.



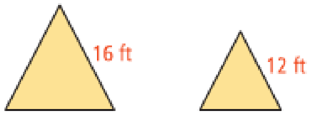
Area of smaller parallelogram $= 6 \text{ in.}^2$

14.



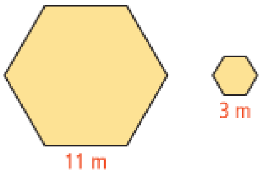
Area of larger trapezoid $= 121 \text{ m}^2$

15.



Area of larger triangle $= 105\ \text{ft}^2$

16.



Area of smaller hexagon $= 23\ \text{m}^2$

17. **Remodeling** The scale factor of the dimensions of two similar wood floors is 4 : 3. It costs \$216 to refinish the smaller wood floor. At that rate, how much would it cost to refinish the larger wood floor?

See Problem 3.

18. **Decorating** An embroidered placemat costs \$3.95. An embroidered tablecloth is similar to the placemat, but four times as long and four times as wide. How much would you expect to pay for the tablecloth?

Find the scale factor and the ratio of perimeters for each pair of similar figures.

See Problem 4.

19. two regular octagons with areas $4\ \text{ft}^2$ and $16\ \text{ft}^2$

20. two triangles with areas $75\ \text{m}^2$ and $12\ \text{m}^2$

21. two trapezoids with areas $49\ \text{cm}^2$ and $9\ \text{cm}^2$

22. two parallelograms with areas $18\ \text{in.}^2$ and $32\ \text{in.}^2$

23. two equilateral triangles with areas $16\sqrt{3}\ \text{ft}^2$ and $\sqrt{3}\ \text{ft}^2$

24. two circles with areas $2\pi\ \text{cm}^2$ and $200\pi\ \text{cm}^2$

B Apply

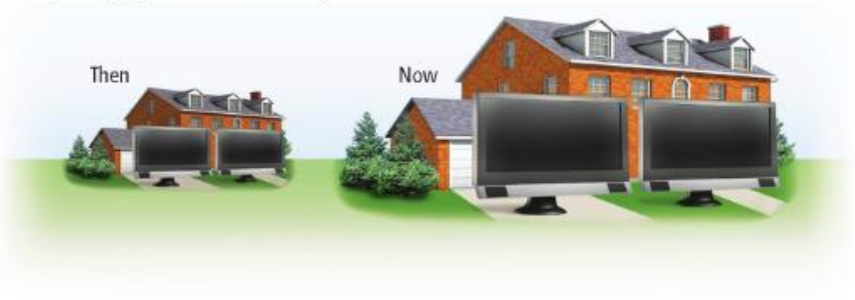
The scale factor of two similar polygons is given. Find the ratio of their perimeters and the ratio of their areas.

25. 3 : 1 **26.** 2 : 5 **27.** $\frac{2}{3}$ **28.** $\frac{7}{4}$ **29.** 6 : 1

30. The area of a regular decagon is $50\ \text{cm}^2$. What is the area of a regular decagon with sides four times the sides of the smaller decagon?

Ⓐ $200\ \text{cm}^2$ Ⓑ $500\ \text{cm}^2$ Ⓒ $800\ \text{cm}^2$ Ⓓ $2000\ \text{cm}^2$

31. **Error Analysis** A reporter used the graphic below to show that the number of houses with more than two televisions had doubled in the past few years. Explain why this graphic is misleading.



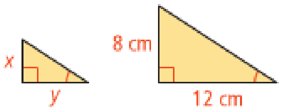
32. **Think About a Plan** Two similar rectangles have areas 27 in.^{2} and 48 in.^{2}. The length of one side of the larger rectangle is 16 in. What are the dimensions of both rectangles?

- How does the ratio of the similar rectangles compare to their scale factor?
- How can you use the dimensions of the larger rectangle to find the dimensions of the smaller rectangle?

33. The longer sides of a parallelogram are 5 m. The longer sides of a similar parallelogram are 15 m. The area of the smaller parallelogram is 28 m^2. What is the area of the larger parallelogram?

Algebra **Find the values of x and y when the smaller triangle shown here has the given area.**

34. 3 cm^2
35. 6 cm^2
36. 12 cm^2
37. 16 cm^2
38. 24 cm^2
39. 48 cm^2



40. **Medicine** For some medical imaging, the scale of the image is 3 : 1. That means that if an image is 3 cm long, the corresponding length on the person's body is 1 cm. Find the actual area of a lesion if its image has area 2.7 cm^2.

41. In $\triangle RST$, $RS = 20$ m, $ST = 25$ m, and $RT = 40$ m.

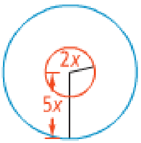
a. **Open-Ended** Choose a convenient scale. Then use a ruler and compass to draw $\triangle R'S'T' \sim \triangle RST$.

b. **Constructions** Construct an altitude of $\triangle R'S'T'$ and measure its length. Find the area of $\triangle R'S'T'$.

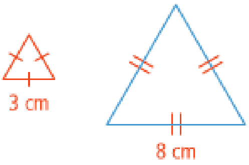
c. **Estimation** Estimate the area of $\triangle RST$.

Compare the blue figure to the red figure. Find the ratios of (a) their perimeters and (b) their areas.

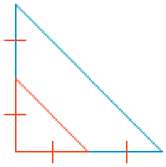
42.



43.

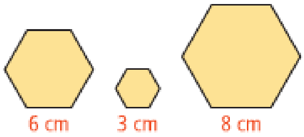


44.



45. **a.** Find the area of a regular hexagon with sides 2 cm long. Leave your answer in simplest radical form.

b. Use your answer to part (a) and Theorem 10-7 to find the areas of the regular hexagons shown at the right.

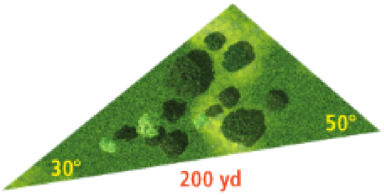


46. **Writing** The enrollment at an elementary school is going to increase from 200 students to 395 students. A parents' group is planning to increase the 100 ft-by-200 ft playground area to a larger area that is 200 ft by 400 ft. What would you tell the parents' group when they ask your opinion about whether the new playground will be large enough?

47. a. Surveying A surveyor measured one side and two angles of a field, as shown in the diagram. Use a ruler and a protractor to draw a similar triangle.

b. Measure the sides and altitude of your triangle and find its perimeter and area.

c. Estimation Estimate the perimeter and area of the field.



Challenge

Reasoning Complete each statement with *always, sometimes,* or *never*. Justify your answers.

48. Two similar rectangles with the same perimeter are __?__ congruent.

49. Two rectangles with the same area are __?__ similar.

50. Two rectangles with the same area and different perimeters are __?__ similar.

51. Similar figures __?__ have the same area.

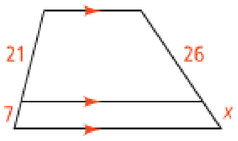
Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

52. Two regular hexagons have sides in the ratio 3 : 5. The area of the smaller hexagon is 81 m^2. In square meters, what is the area of the larger hexagon?

53. What is the value of x in the diagram at the right?



54. A trapezoid has base lengths of 9 in. and 4 in. and a height of 3 in. What is the area of the trapezoid in square inches?

55. In quadrilateral $ABCD$, $m\angle A = 62$, $m\angle B = 101$, and $m\angle C = 42$. What is $m\angle D$?

Mixed Review

Find the area of each regular polygon. **See Lesson 10-3.**

56. a square with a 5-cm radius

57. a pentagon with apothem 13.8 and side length 20

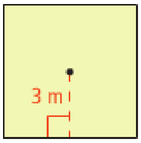
58. an octagon with apothem 12 and side length 10

59. An angle bisector divides the opposite side of a triangle into segments 4 cm and 6 cm long. A second side of the triangle is 8 cm long. What are all possible lengths for the third side of the triangle? **See Lesson 7-5.**

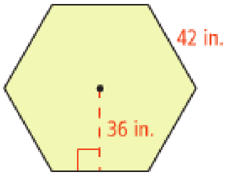
Get Ready! **To prepare for Lesson 10-5, do Exercises 60–62.**

Find the area of each regular polygon. **See Lesson 10-3.**

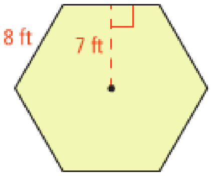
60.



61.



62.



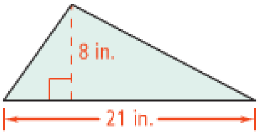
10 Mid-Chapter Quiz

MathXL® for School
Go to PowerGeometry.com

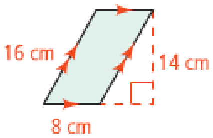
Do you know HOW?

Find the area of each figure.

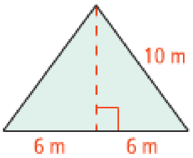
1.



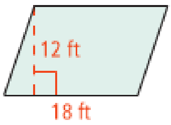
2.



3.



4.



5. What is the area of a parallelogram with a base of 17 in. and a corresponding height of 12 in.?

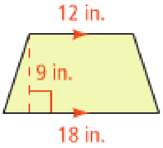
6. If the base of a triangle is 10 cm, and its area is 35 cm^2, what is the height of the triangle?

7. The area of a parallelogram is 36 in.^2, and its height is 3 in. How long is the corresponding base?

8. An equilateral triangle has a perimeter of 60 m and a height of 17.3 m. What is its area?

Find the area of each figure.

9.



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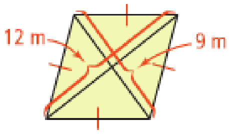

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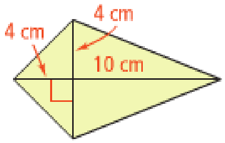
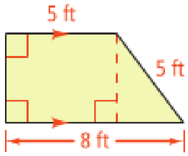


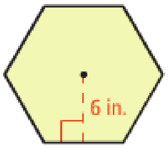
12.



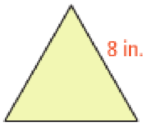
13. The area of a trapezoid is 100 ft^2. The sum of its two bases is 25 ft. What is the height of the trapezoid?

Find the area of each regular polygon. Round your answer to the nearest tenth.

14.



15.



16. A regular octagon has sides 15 cm long. The apothem is 18.5 cm long. What is the area of the octagon?

17. The radius of a regular hexagon is 5 ft. What is the area of the hexagon to the nearest square foot?

The scale factor of $\triangle ABC$ to $\triangle DEF$ is 3 : 5. Fill in the missing information.

18. The perimeter of $\triangle ABC$ is 36 in.
The perimeter of $\triangle DEF$ is $\underline{\ ?\ }$.

19. The area of $\triangle ABC$ is $\underline{\ ?\ }$.
The area of $\triangle DEF$ is 125 in.^2.

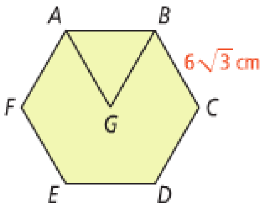
20. The areas of two similar triangles are 1.44 and 1.00. Find their scale factor.

21. The ratio of the perimeters of two similar triangles is 1 : 3. The area of the larger triangle is 27 ft^2. What is the area of the smaller triangle?

Do you UNDERSTAND?

22. **Open-Ended** Draw a rhombus. Measure the lengths of the diagonals. What is the area?

23. **Writing** Describe two different methods for finding the area of regular hexagon *ABCDEF*. What is the area?



24. **Reasoning** Suppose the diagonals of a kite are doubled. How does this affect its area? Explain.

10-5 Trigonometry and Area

Objective To find areas of regular polygons and triangles using trigonometry

Use techniques you've already learned to find the height of the triangle.

SOLVE IT! **Getting Ready!**

The pennant at the right is in the shape of an isosceles triangle. The measure of the vertex angle is 20. What is the area of the pennant? How do you know?

In this lesson you will use isosceles triangles and trigonometry to find the area of a regular polygon.

Essential Understanding You can use trigonometry to find the area of a regular polygon when you know the length of a side, radius, or apothem.

Problem 1 Finding Area

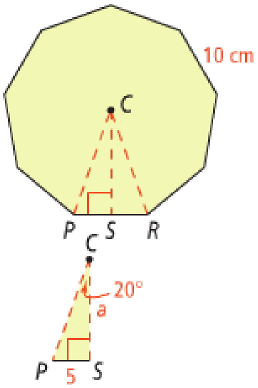
What is the area of a regular nonagon with 10-cm sides?

Think

What is the apothem in the diagram?
The apothem is the altitude of the isosceles triangle. The apothem bisects the central angle and the side of the polygon.

Draw a regular nonagon with center C. Draw $\overline{CP}$ and $\overline{CR}$ to form isosceles $\triangle PCR$. The measure of central $\angle PCR$ is $\frac{360}{9}$, or 40. The perimeter is $9 \cdot 10$, or 90 cm. Draw the apothem $\overline{CS}$.

$m\angle PCS = \frac{1}{2}m\angle PCR = 20$ and $PS = \frac{1}{2}PR = 5$ cm.



Let a represent CS. Find a and substitute into the area formula.

$\tan 20^\circ = \frac{5}{a}$ Use the tangent ratio.

$a = \frac{5}{\tan 20^\circ}$ Solve for a.

$A = \frac{1}{2}ap$

$= \frac{1}{2} \cdot \frac{5}{\tan 20^\circ} \cdot 90$ Substitute $\frac{5}{\tan 20^\circ}$ for a and 90 for p.

≈ 618.1824194 Use a calculator.

The area of the regular nonagon is about 618 cm^2.

Got It? **1.** What is the area of a regular pentagon with 4-in. sides? Round your answer to the nearest square inch.

Problem 2 Finding Area

GRIDDED RESPONSE

Road Signs A stop sign is a regular octagon. The standard size has a 16.2-in. radius. What is the area of the stop sign to the nearest square inch?

Know

The radius and the number of sides of the octagon

Need

The apothem and the length of a side

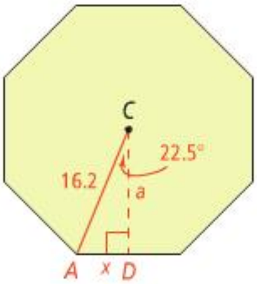
Plan

Use trigonometric ratios to find the apothem and the length of a side

Step 1 Let a represent the apothem. Use the cosine ratio to find a. The measure of a central angle of the octagon is $\frac{360}{8}$, or 45. So $m\angle C = \frac{1}{2}(45) = 22.5$.

$\cos 22.5^\circ = \frac{a}{16.2}$ Use the cosine ratio.

$16.2(\cos 22.5^\circ) = a$ Multiply each side by 16.2.



Step 2 Let x represent AD. Use the sine ratio to find x.

$\sin 22.5^\circ = \frac{x}{16.2}$ Use the sine ratio.

$16.2(\sin 22.5^\circ) = x$ Multiply each side by 16.2.

Step 3 Find the perimeter of the octagon.

$p = 8 \cdot \text{length of a side}$

$= 8 \cdot 2x$ The length of each side is $2x$.

$= 8 \cdot 2 \cdot 16.2(\sin 22.5^\circ)$ Substitute for x.

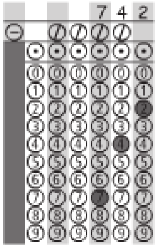
$= 259.2(\sin 22.5^\circ)$ Simplify.

Step 4 Substitute into the area formula.

$A = \frac{1}{2}ap$

$= \frac{1}{2} \cdot 16.2(\cos 22.5^\circ) \cdot 259.2(\sin 22.5^\circ)$ Substitute for a and p.

≈ 742.2924146 Use a calculator.



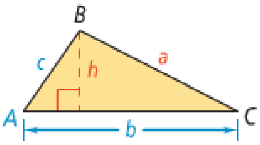
The area of the stop sign is about 742 in.^{2}.

Got It? **2. a.** A tabletop has the shape of a regular decagon with a radius of 9.5 in. What is the area of the tabletop to the nearest square inch?

b. Reasoning Suppose the radius of a regular polygon is doubled. How does the area of the polygon change? Explain.

Essential Understanding You can use trigonometry to find the area of a triangle when you know the length of two sides and the included angle.

Suppose you want to find the area of $\triangle ABC$, but you know only $m\angle A$ and the length b and c. To use the formula $A = \frac{1}{2}bh$, you need to know the height. You can find the height by using the sine ratio.



$\sin A = \frac{h}{c}$ Use the sine ratio.

$h = c(\sin A)$ Solve for h.

Now substitute for h in the formula $Area = \frac{1}{2}bh$.

$$Area = \frac{1}{2}bc(\sin A)$$

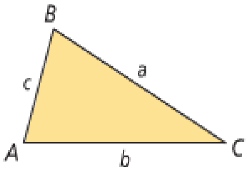
This completes the proof of the following theorem for the case in which $\angle A$ is acute.

take note

Theorem 10-8 Area of a Triangle Given SAS

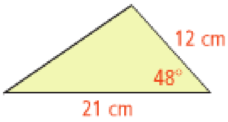
The area of a triangle is half the product of the lengths of two sides and the sine of the included angle.

$$\text{Area of } \triangle ABC = \frac{1}{2}bc(\sin A)$$



Problem 3 Finding Area

What is the area of the triangle?



$$\text{Area} = \frac{1}{2} \cdot \text{side length} \cdot \text{side length} \cdot \text{sine of included angle}$$

$= \frac{1}{2} \cdot 12 \cdot 21 \cdot \sin 48^\circ$ Substitute.

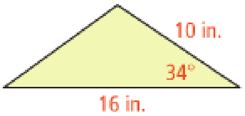
≈ 93.63624801 Use a calculator.

The area of the triangle is about 94 cm^2.

Plan

Which formula should you use?
The diagram gives the lengths of two sides and the measure of the included angle. Use the formula for the area of a triangle given SAS.

Got It? **3.** What is the area of the triangle? Round your answer to the nearest square inch.

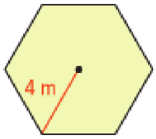


Lesson Check

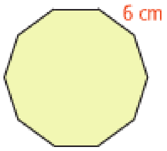
Do you know HOW?

What is the area of each regular polygon? Round your answers to the nearest tenth.

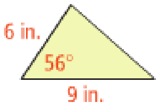
1.



2.



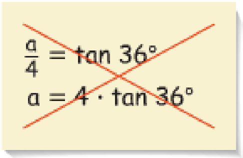
3. What is the area of the triangle at the right to the nearest square inch?



Do you UNDERSTAND?

4. Reasoning A diagonal through the center of a regular hexagon is 12 cm long. Is it possible to find the area of this hexagon? Explain.

5. Error Analysis Your classmate needs to find the area of a regular pentagon with 8-cm sides. To find the apothem, he sets up and solves a trigonometric ratio. What error did he make? Explain.



$$\xcancel{\begin{aligned}\frac{a}{4} &= \tan 36^\circ\\ a &= 4 \cdot \tan 36^\circ\end{aligned}}$$

Practice and Problem-Solving Exercises

A Practice

Find the area of each regular polygon. Round your answers to the nearest tenth. **See Problems 1 and 2.**

6. octagon with side length 6 cm

7. decagon with side length 4 yd

8. pentagon with radius 3 ft

9. nonagon with radius 7 in.

10. dodecagon with radius 20 cm

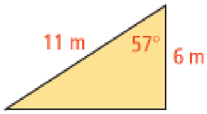
11. 20-gon with radius 2 mm

12. 18-gon with perimeter 72 mm

13. 15-gon with perimeter 180 cm

Find the area of each triangle. Round your answers to the nearest tenth. **See Problem 3.**

14.



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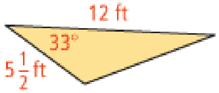

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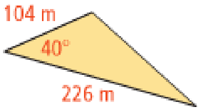

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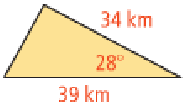

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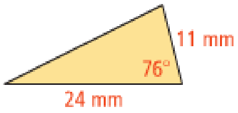

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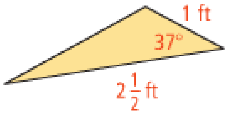


B Apply

20. *PQRST* is a regular pentagon with center *O* and radius 10 in. Find each measure. If necessary, round your answers to the nearest tenth.

a. $m\angle POQ$

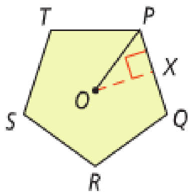
b. $m\angle POX$

c. OX

d. PQ

e. perimeter of *PQRST*

f. area of *PQRST*



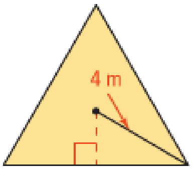
21. **Writing** Describe three ways to find the area of a regular hexagon if you know only the length of a side.

22. **Think About a Plan** The surveyed lengths of two adjacent sides of a triangular plot of land are 80 yd and 150 yd. The angle between the sides is 67°. What is the area of the parcel of land to the nearest square yard?

- Can you *draw a diagram* to represent the situation?
- Which formula for the area of a triangle should you use?

Find the perimeter and area of each regular polygon to the nearest tenth.

23.



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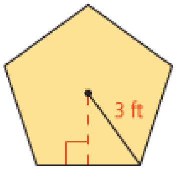

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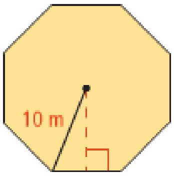

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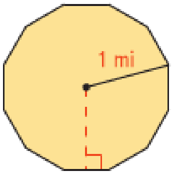
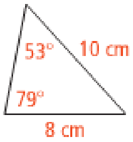


27. **Architecture** The Pentagon in Arlington, Virginia, is one of the world's largest office buildings. It is a regular pentagon, and the length of each of its sides is 921 ft. What is the area of land that the Pentagon covers to the nearest thousand square feet?

28. What is the area of the triangle shown at the right?



29. The central angle of a regular polygon is 10°. The perimeter of the polygon is 108 cm. What is the area of the polygon?

30. Replacement glass for energy-efficient windows costs $5/ft^2. About how much will you pay for replacement glass for a regular hexagonal window with a radius of 2 ft?

Ⓐ $10.39 Ⓑ $27.78 Ⓒ $45.98 Ⓓ $51.96

Regular polygons A and B are similar. Compare their areas.

31. The apothem of Pentagon A equals the radius of Pentagon B.

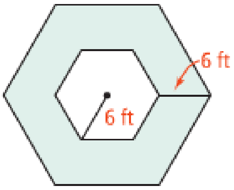
32. The length of a side of Hexagon A equals the radius of Hexagon B.

33. The radius of Octagon A equals the apothem of Octagon B.

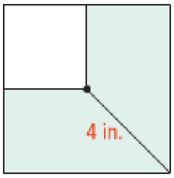
34. The perimeter of Decagon A equals the length of a side of Decagon B.

The polygons are regular polygons. Find the area of the shaded region.

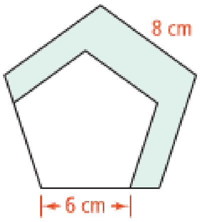
35.



36.



37.



Challenge

38. **Surveying** A surveyor wants to mark off a triangular parcel with an area of 1 acre (1 acre $= 43{,}560$ ft^2). One side of the triangle extends 300 ft along a straight road. A second side extends at an angle of 65° from one end of the first side. What is the length of the second side to the nearest foot?

39. Segments are drawn between the midpoints of consecutive sides of a regular pentagon to form another regular pentagon. Find, to the nearest hundredth, the ratio of the area of the smaller pentagon to the area of the larger pentagon.

Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

40. A regular polygon has a perimeter of 54 m and an apothem of $3\sqrt{3}$ m. What is the area of the polygon to the nearest tenth of a square meter?

41. The legs of a right triangle have lengths of 8 in. and 15 in. What is the length of the hypotenuse in inches?

42. $\triangle PEN \cong \triangle LIV$. If $m\angle P = 36$ and $m\angle N = 82$, what is $m\angle I$?

43. The perimeter of a parallelogram is 23.6 ft. If its length and width are doubled, what is the perimeter of the parallelogram in feet?

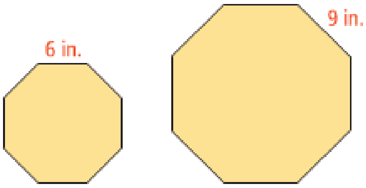
44. The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments of lengths 8 and 10. What is the length of the shorter leg of the triangle?

Mixed Review

45. Two regular octagons are shown.

a. What is the scale factor of the smaller octagon to the larger octagon?

b. The area of the larger octagon is 391.1 in.^{2}. What is the area of the smaller octagon to the nearest tenth of a square inch?



See Lesson 10-4.

For Exercises 46–49, use vectors $\vec{a}$ and $\vec{c}$ at the right.

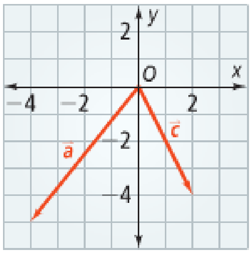
46. What is the sum of $\vec{a}$ and $\vec{c}$ as an ordered pair?

47. Describe a vector $\vec{d}$ such that $\vec{a} + \vec{d} = \vec{c}$.

48. Describe a vector $\vec{e}$ such that $\vec{c} + \vec{e} = \vec{a}$.

49. Which two vectors have $\vec{c}$ as their sum?

$\langle -2, -2 \rangle$ $\langle 2, -4 \rangle$ $\langle 4, -2 \rangle$



See Lesson 8-5.

Get Ready! **To prepare for Lesson 10-6, do Exercises 50–52.**

Find the diameter or radius of each circle.

See Lesson 1-8.

50. $r = 7$ cm, $d =$ ■

51. $d = 5$ in., $r =$ ■

52. $r = 1.6$ m, $d =$ ■

10-6 Circles and Arcs

Objectives To find the measures of central angles and arcs
To find the circumference and arc length

Whoa—that wheel has to complete over a thousand rotations to go one mile.

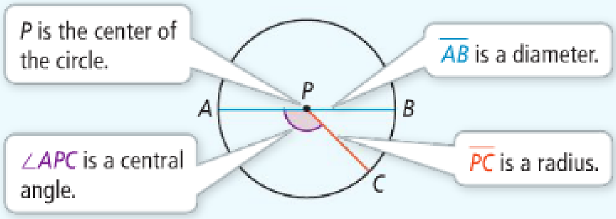
SOLVE IT! **Getting Ready!**

The bicycle wheel shown at the right travels 63 in. in one complete rotation. If the wheel rotates only 120° about the center, how far does it travel? Justify your reasoning.

Lesson Vocabulary
- circle
- center
- diameter
- radius
- congruent circles
- central angle
- semicircle
- minor arc
- major arc
- adjacent arcs
- circumference
- pi
- concentric circles
- arc length
- congruent arcs

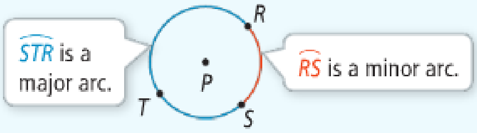
In a plane, a **circle** is the set of all points equidistant from a given point called the **center.** You name a circle by its center. Circle P ($\odot P$) is shown below.

A **diameter** is a segment that contains the center of a circle and has both endpoints on the circle. A **radius** is a segment that has one endpoint at the center and the other endpoint on the circle. **Congruent circles** have congruent radii. A **central angle** is an angle whose vertex is the center of the circle.


P is the center of the circle.
$\overline{AB}$ is a diameter.
$\angle APC$ is a central angle.
$\overline{PC}$ is a radius.
A
P
B
C

Essential Understanding You can find the length of part of a circle's circumference by relating it to an angle in the circle.

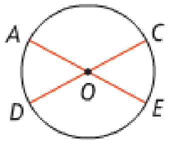
An arc is a part of a circle. One type of arc, a **semicircle,** is half of a circle. A **minor arc** is smaller than a semicircle. A **major arc** is larger than a semicircle. You name a minor arc by its endpoints and a major arc or a semicircle by its endpoints and another point on the arc.


$\overset{\frown}{STR}$ is a major arc.
$\overset{\frown}{RS}$ is a minor arc.
R
P
T
S

Plan

How can you identify the minor arcs?
Since a minor arc contains all the points in the interior of a central angle, start by identifying the central angles in the diagram.

Problem 1 Naming Arcs



A What are the minor arcs of ⊙O?

The minor arcs are $\widehat{AD}$, $\widehat{CE}$, $\widehat{AC}$, and $\widehat{DE}$.

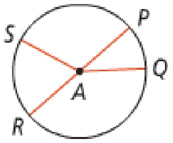
B What are the semicircles of ⊙O?

The semicircles are $\widehat{ACE}$, $\widehat{CED}$, $\widehat{EDA}$, and $\widehat{DAC}$.

C What are the major arcs of ⊙O that contain point A?

The major arcs that contain point A are $\widehat{ACD}$, $\widehat{CEA}$, $\widehat{EDC}$, and $\widehat{DAE}$.

Got It? **1. a.** What are the minor arcs of ⊙A?
b. What are the semicircles of ⊙A?
c. What are the major arcs of ⊙A that contain point Q?



take note

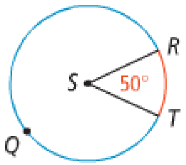
Key Concept Arc Measure

Arc Measure
The measure of a minor arc is equal to the measure of its corresponding central angle.

The measure of a major arc is the measure of the related minor arc subtracted from 360.

The measure of a semicircle is 180.

Example



$m\widehat{RT} = m\angle RST = 50$
$m\widehat{TQR} = 360 - m\widehat{RT}$
$= 310$

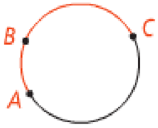
Adjacent arcs are arcs of the same circle that have exactly one point in common. You can add the measures of adjacent arcs just as you can add the measures of adjacent angles.

take note

Postulate 10-2 Arc Addition Postulate

The measure of the arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$$m\widehat{ABC} = m\widehat{AB} + m\widehat{BC}$$



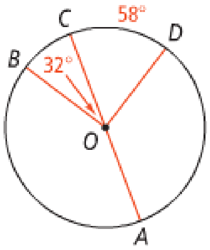
Think

How can you find $m\widehat{BD}$?

$\widehat{BD}$ is formed by adjacent arcs $\widehat{BC}$ and $\widehat{CD}$. Use the Arc Addition Postulate.

Problem 2 Finding the Measures of Arcs

What is the measure of each arc in $\odot O$?



A $\widehat{BC}$ $\quad m\widehat{BC} = m\angle BOC = 32$

B $\widehat{BD}$ $\quad m\widehat{BD} = m\widehat{BC} + m\widehat{CD}$

$m\widehat{BD} = 32 + 58 = 90$

C $\widehat{ABC}$ $\quad \widehat{ABC}$ is a semicircle.

$m\widehat{ABC} = 180$

D $\widehat{AB}$ $\quad m\widehat{AB} = 180 - 32 = 148$

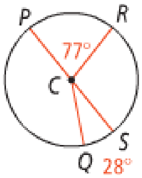
Got It? **2.** What is the measure of each arc in $\odot C$?

a. $m\widehat{PR}$

b. $m\widehat{RS}$

c. $m\widehat{PRQ}$

d. $m\widehat{PQR}$



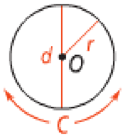
The **circumference** of a circle is the distance around the circle. The number **pi** (π) is the ratio of the circumference of a circle to its diameter.

take note

Theorem 10-9 Circumference of a Circle

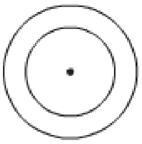
The circumference of a circle is π times the diameter.

$C = \pi d$ or $C = 2\pi r$



The number π is irrational, so you cannot write it as a terminating or repeating decimal. To approximate π, you can use 3.14, $\frac{22}{7}$, or the π key on your calculator.

Coplanar circles that have the same center are **concentric circles.**



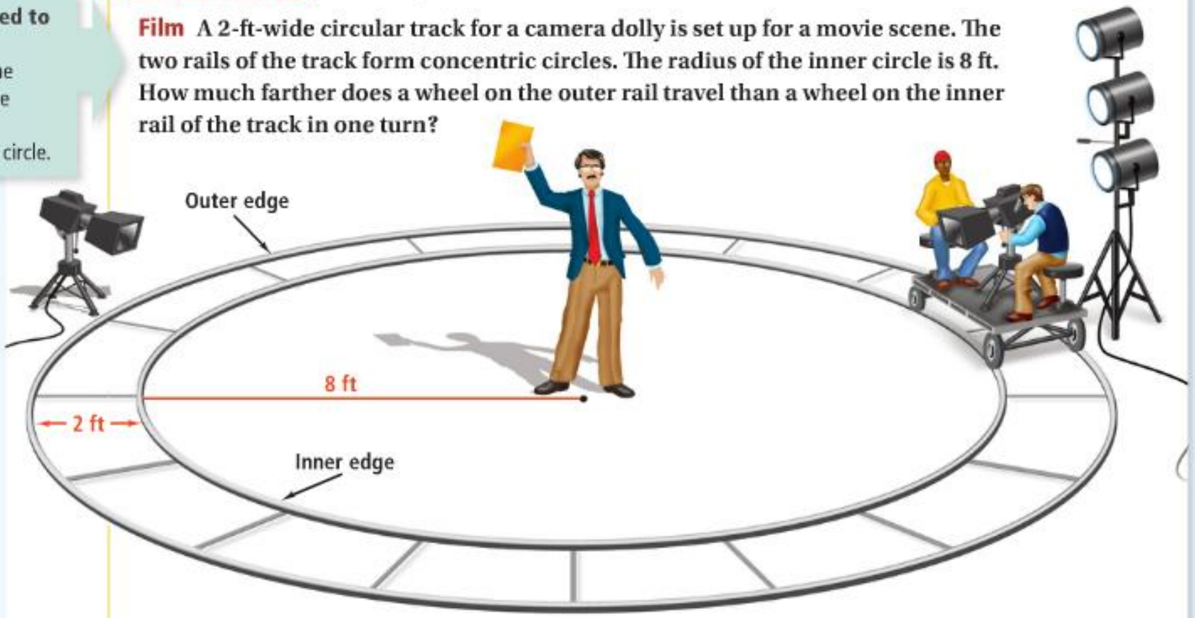
Concentric circles

Plan

What do you need to find?
You need to find the distance around the track, which is the circumference of a circle.

Problem 3 Finding a Distance

Film **A 2-ft-wide circular track for a camera dolly is set up for a movie scene. The two rails of the track form concentric circles. The radius of the inner circle is 8 ft. How much farther does a wheel on the outer rail travel than a wheel on the inner rail of the track in one turn?**



circumference of inner circle $= 2\pi r$ Use the formula for the circumference of a circle.

$= 2\pi(8)$ Substitute 8 for *r*.

$= 16\pi$ Simplify.

The radius of the outer circle is the radius of the inner circle plus the width of the track.

radius of the outer circle $= 8 + 2 = 10$

circumference of outer circle $= 2\pi r$ Use the formula for the circumference of a circle.

$= 2\pi(10)$ Substitute 10 for *r*.

$= 20\pi$ Simplify.

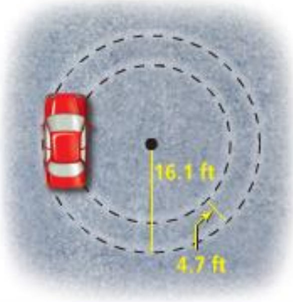
The difference in the two distances traveled is $20\pi - 16\pi$, or 4π ft.

$4\pi \approx 12.56637061$ Use a calculator.

A wheel on the outer edge of the track travels about 13 ft farther than a wheel on the inner edge of the track.

Got It? **3. a.** A car has a circular turning radius of 16.1 ft. The distance between the two front tires is 4.7 ft. How much farther does a tire on the outside of the turn travel than a tire on the inside?

b. Reasoning Suppose the radius of $\odot A$ is equal to the diameter of $\odot B$. What is the ratio of the circumference of $\odot A$ to the circumference of $\odot B$? Explain.



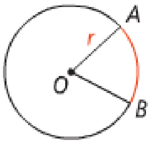
The measure of an arc is in degrees while the **arc length** is a fraction of a circle's circumference. An arc of 60° represents $\frac{60}{360}$ or $\frac{1}{6}$ of the circle. Its arc length is $\frac{1}{6}$ the circumference of the circle. This observation suggests the following theorem.

take note

Theorem 10-10 Arc Length

The length of an arc of a circle is the product of the ratio $\frac{\text{measure of the arc}}{360}$ and the circumference of the circle.

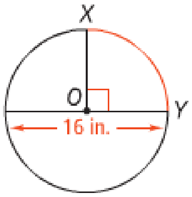
$$\text{length of } \widehat{AB} = \frac{m\widehat{AB}}{360} \cdot 2\pi r$$
$$= \frac{m\widehat{AB}}{360} \cdot \pi d$$



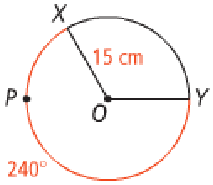
Problem 4 Finding Arc Length

What is the length of each arc shown in red? Leave your answer in terms of π.

A



B



Think

How do you know which formula to use?
It depends on whether the diameter is given or the radius is given.

$$\text{length of } \widehat{XY} = \frac{m\widehat{XY}}{360} \cdot \pi d$$ Use a formula for arc length.

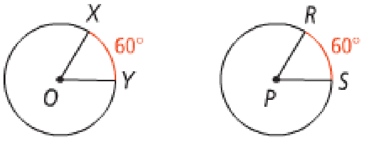
$$= \frac{90}{360} \cdot \pi(16)$$ Substitute.

$$= 4\pi \text{ in.}$$ Simplify.

$$\text{length of } \widehat{XPY} = \frac{m\widehat{XPY}}{360} \cdot 2\pi r$$
$$= \frac{240}{360} \cdot 2\pi(15)$$
$$= 20\pi \text{ cm}$$

Got It? **4.** What is the length of a semicircle with radius 1.3 m? Leave your answer in terms of π.

It is possible for two arcs of different circles to have the same measure but different lengths. It is also possible for two arcs of different circles to have the same length but different measures. **Congruent arcs** are arcs that have the same measure *and* are in the same circle or in congruent circles.



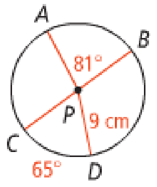
$\odot O \cong \odot P$

$\widehat{XY} \cong \widehat{RS}$

Lesson Check

Do you know HOW?

Use ⊙P at the right to answer each question. For Exercises 5 and 6, leave your answers in terms of π.



1. What is the name of a minor arc?

2. What is the name of a major arc?

3. What is the name of a semicircle?

4. What is $m\overset{\frown}{AB}$?

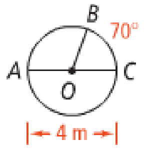
5. What is the circumference of ⊙P?

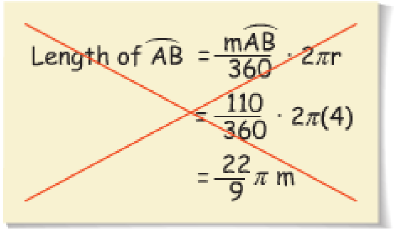
6 What is the length of $\overset{\frown}{BD}$?

Do you UNDERSTAND?

7. Vocabulary What is the difference between the measure of an arc and arc length? Explain.

8. Error Analysis Your class must find the length of $\overset{\frown}{AB}$. A classmate submits the following solution. What is the error?



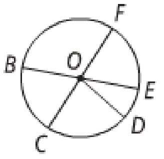


$$\text{Length of } \overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360} \cdot 2\pi r$$
$$= \frac{110}{360} \cdot 2\pi(4)$$
$$= \frac{22}{9}\pi \text{ m}$$

Practice and Problem-Solving Exercises

A Practice

Name the following in ⊙O. **See Problem 1.**

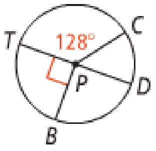


9. the minor arcs

10. the major arcs

11. the semicircles

Find the measure of each arc in ⊙P. **See Problem 2.**



12. $\overset{\frown}{TC}$ **13.** $\overset{\frown}{TBD}$ **14.** $\overset{\frown}{BTC}$

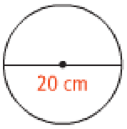
15. $\overset{\frown}{TCB}$ **16.** $\overset{\frown}{CD}$ **17.** $\overset{\frown}{CBD}$

18. $\overset{\frown}{TCD}$ **19.** $\overset{\frown}{DB}$ **20.** $\overset{\frown}{TDC}$

21. $\overset{\frown}{TB}$ **22.** $\overset{\frown}{BC}$ **23.** $\overset{\frown}{BCD}$

Find the circumference of each circle. Leave your answer in terms of π. **See Problem 3.**

24.



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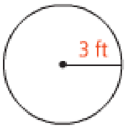

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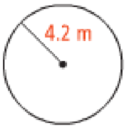

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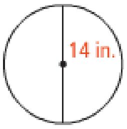


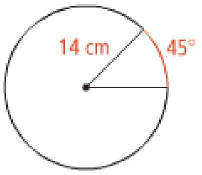
28. The camera dolly track in Problem 3 can be expanded so that the diameter of the outer circle is 70 ft. How much farther will a wheel on the outer rail travel during one turn around the track than a wheel on the inner rail?

29. The wheel of a compact car has a 25-in. diameter. The wheel of a pickup truck has a 31-in. diameter. To the nearest inch, how much farther does the pickup truck wheel travel in one revolution than the compact car wheel?

Find the length of each arc shown in red. Leave your answer in terms of π.

See Problem 4.

30.



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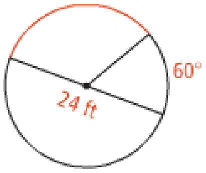

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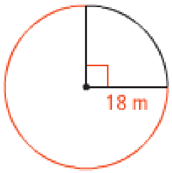

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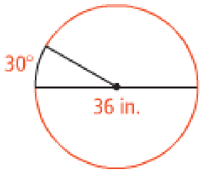

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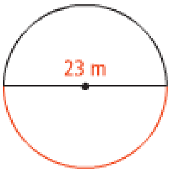

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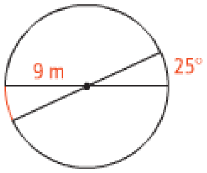
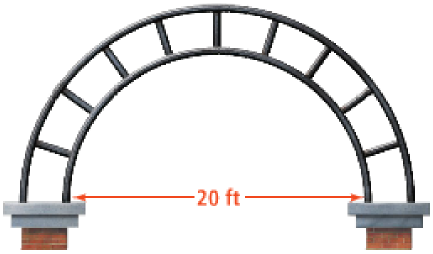


B Apply

36. **Think About a Plan** Nina designed a semicircular arch made of wrought iron for the top of a mall entrance. The nine segments between the two concentric semicircles are each 3 ft long. What is the total length of wrought iron used to make this structure? Round your answer to the nearest foot.

- What do you know from the diagram?
- What formula should you use to find the amount of wrought iron used in the semicircular arches?



Find each indicated measure for ⊙*O*.

37. $m\angle EOF$

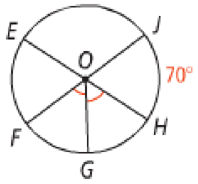
38. $m\overset{\frown}{EJH}$

39. $m\overset{\frown}{FH}$

40. $m\angle FOG$

41. $m\overset{\frown}{JEG}$

42. $m\overset{\frown}{HFJ}$



43. **Pets** A hamster wheel has a 7-in. diameter. How many feet will a hamster travel in 100 revolutions of the wheel?

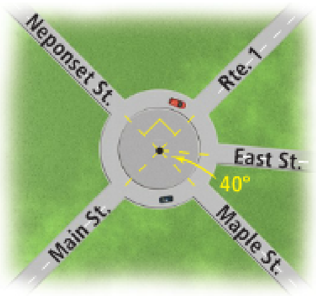
44. **Traffic** Five streets come together at a traffic circle, as shown at the right. The diameter of the circle traveled by a car is 200 ft. If traffic travels counterclockwise, what is the approximate distance from East St. to Neponset St.?

Ⓐ 227 ft

Ⓑ 244 ft

Ⓒ 454 ft

Ⓓ 488 ft


Neponset St.
Rte. 1
East St.
40°
Main St.
Maple St.

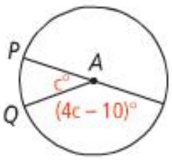
45. **Writing** Describe two ways to find the arc length of a major arc if you are given the measure of the corresponding minor arc and the radius of the circle.

46. **Time** Hands of a clock suggest an angle whose measure is continually changing. How many degrees does a minute hand move through during each time interval?

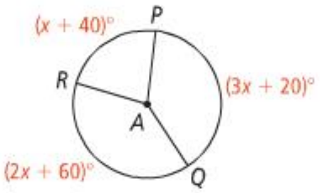
a. 1 min **b.** 5 min **c.** 20 min

Algebra **Find the value of each variable.**

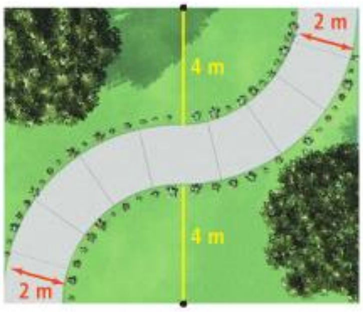
47.



48.



49. **Landscape Design** A landscape architect is constructing a curved path through a rectangular yard. The curved path consists of two 90° arcs. He plans to edge the two sides of the path with plastic edging. What is the total length of plastic edging he will need? Round your answer to the nearest meter.

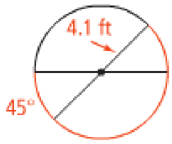


50. **Reasoning** Suppose the radius of a circle is doubled. How does this affect the circumference of the circle? Explain.

51. A 60° arc of ⊙A has the same length as a 45° arc of ⊙B. What is the ratio of the radius of ⊙A to the radius of ⊙B?

Find the length of each arc shown in red. Leave your answer in terms of π.

52.



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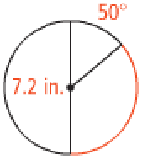

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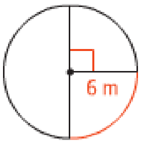
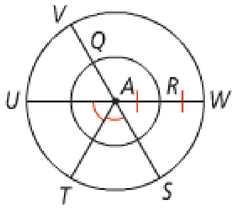


55. **Coordinate Geometry** Find the length of a semicircle with endpoints (1, 3) and (4, 7). Round your answer to the nearest tenth.

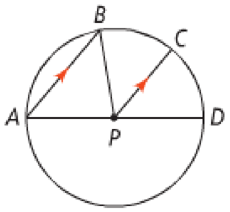
56. In ⊙O, the length of $\widehat{AB}$ is 6π cm and $m\widehat{AB}$ is 120. What is the diameter of ⊙O?

Challenge

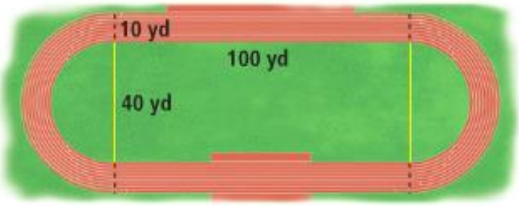
57. The diagram below shows two concentric circles. $\overline{AR} \cong \overline{RW}$. Show that the length of $\widehat{ST}$ is equal to the length of $\widehat{QR}$.



58. **Given:** ⊙P with $\overline{AB} \parallel \overline{PC}$
Proof **Prove:** $m\widehat{BC} = m\widehat{CD}$



59. Sports An athletic field is a 100 yd-by-40 yd rectangle, with a semicircle at each of the short sides. A running track 10 yd wide surrounds the field. If the track is divided into eight lanes of equal width, what is the distance around the track along the inside edge of each lane?



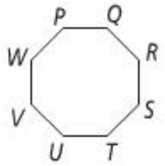
Standardized Test Prep

SAT/ACT

60. The radius of a circle is 12 cm. What is the length of a 60° arc?

Ⓐ 3π cm Ⓑ 4π cm Ⓒ 5π cm Ⓓ 6π cm

61. What is the image of *P* for a 135° clockwise rotation about the center of the regular octagon?



Ⓕ *S* Ⓗ *U*

Ⓖ *T* Ⓘ *R*

62. Which of the following are the sides of a right triangle?

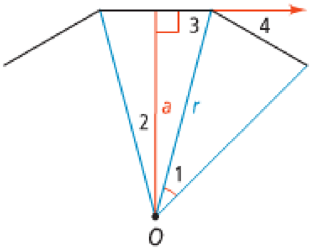
Ⓐ 6, 8, 12 Ⓑ 8, 15, 17 Ⓒ 9, 11, 23 Ⓓ 5, 12, 15

Extended Response

63. Quadrilateral *ABCD* has vertices *A*(1, 1), *B*(4, 1), *C*(4, 6), and *D*(1, 6). Quadrilateral *RSTV* has vertices *R*(−3, 4), *S*(−3, −2), *T*(−13 −2), and *V*(−13, 4). Show that *ABCD* and *RSTV* are similar rectangles.

Mixed Review

Part of a regular dodecagon is shown at the right. **See Lesson 10-5.**



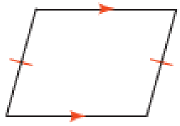
64. What is the measure of each numbered angle?

65. The radius is 19.3 mm. What is the apothem?

66. What is the perimeter and area of the dodecagon to the nearest millimeter or square millimeter?

Can you conclude that the figure is a parallelogram? Explain. **See Lesson 6-3.**

67.



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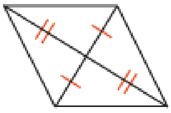

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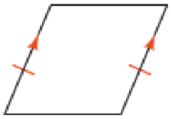


Get Ready! **To prepare for Lesson 10-7, do Exercises 70 and 71.** **See Lesson 10-6.**

70. What is the circumference of a circle with diameter 17 in.?

71. What is the length of a 90° arc in a circle with radius 6 cm?

Concept Byte

Use With Lesson 10-6

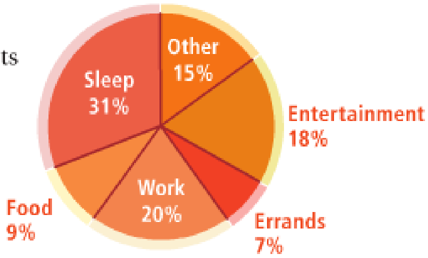
ACTIVITY

Circle Graphs

Circle graphs show data as percents or fractions of a whole. The total of the data must be 100% or 1. The measure of the central angle for a particular category of the data is proportional to the percent or fraction of the total that the category represents. The measures of the central angles in a circle graph have a total of 360. To find a central angle for a category of data, you multiply the percent or fraction that the category represents by 360.

Activity 1

The circle graph at the right shows the results of a time study in which participants recorded how they spent their time over a 24-h period.



1. What is the measure of the central angle that represents the time spent on each activity? Round to the nearest tenth where necessary.

a. Sleep **b.** Food **c.** Work
d. Errands **e.** Entertainment **f.** Other

To make a circle graph, use a compass to make the circle and use a protractor to measure the central angles.

Activity 2

The physical education department asked students which of three sports was their first choice to include in the spring physical education classes. The table at the right shows the results of the survey.

Sport	First Choice
Volleyball	30%
Basketball	25%
Tennis	45%

2. For a circle graph, what is the measure of the central angle for each sport?

3. Use the data to draw and label a circle graph.

Activity 3

A store that sells music CDs keeps track of their weekly sales for inventory purposes. The table at the right shows the sales for the first week in March.

Music Genre	Sales (dollars)
Rock	3150
Country	1800
Rap	2250
Classical	1350
Other	450

4. What percent of the total is each type of music?

5. For a circle graph of the data, what is the measure of the central angle for each type of music?

6. Use the data to draw and label a circle graph.

Concept Byte

Use With Lesson 10-7

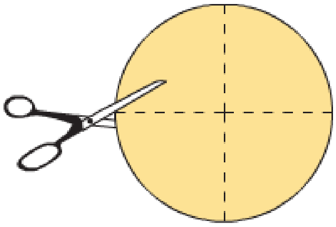
ACTIVITY

Exploring the Area of a Circle

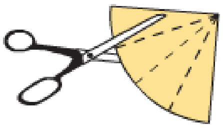
You can use transformations to find the formula for the area of a circle.

Activity

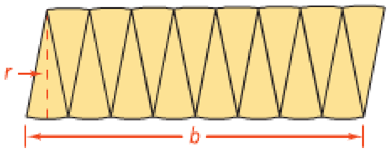
Step 1 Use a compass to draw a large circle. Fold the circle horizontally and then vertically. Cut the circle into four wedges on the fold lines.



Step 2 Fold each wedge into quarters. Cut each wedge on the fold lines. You will have 16 wedges.



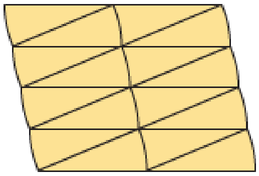
Step 3 Tape the wedges to a piece of paper to form the figure shown at the right. The figure resembles a parallelogram.



1. How does the area of the parallelogram compare with the area of the circle?
2. The base of the parallelogram is formed by arcs of the circle. Explain how the length b relates to the circumference C of the circle.
3. Explain how the length b relates to the radius r of the circle.
4. Write an expression for the area of the parallelogram in terms of r to write a formula for the area of a circle.

Exercises

Repeat Steps 1 and 2 from the activity. Tape the wedges to a piece of paper to form another figure that resembles a parallelogram, as shown at the right.

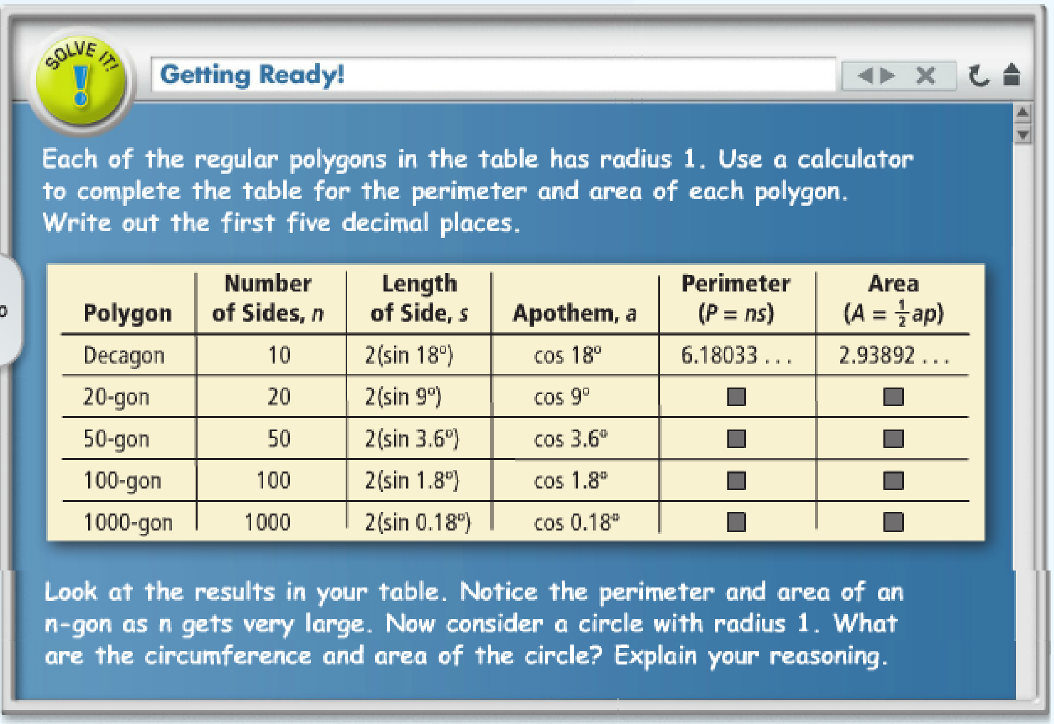


5. What are the base and height of the figure in terms of r?
6. Write an expression for the area of the figure to write a formula for the area of a circle. Is this expression the same as the one you wrote in the activity?

10-7 Areas of Circles and Sectors

Objective To find the areas of circles, sectors, and segments of circles

A regular polygon has a radius and so does a circle.



Getting Ready!

Each of the regular polygons in the table has radius 1. Use a calculator to complete the table for the perimeter and area of each polygon. Write out the first five decimal places.

Polygon	Number of Sides, n	Length of Side, s	Apothem, a	Perimeter ($P = ns$)	Area ($A = \frac{1}{2}ap$)
Decagon	10	2(sin 18°)	cos 18°	6.18033 . . .	2.93892 . . .
20-gon	20	2(sin 9°)	cos 9°	■	■
50-gon	50	2(sin 3.6°)	cos 3.6°	■	■
100-gon	100	2(sin 1.8°)	cos 1.8°	■	■
1000-gon	1000	2(sin 0.18°)	cos 0.18°	■	■

Look at the results in your table. Notice the perimeter and area of an n-gon as n gets very large. Now consider a circle with radius 1. What are the circumference and area of the circle? Explain your reasoning.

Lesson Vocabulary

- sector of a circle
- segment of a circle

In the Solve It, you explored the area of a circle.

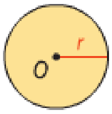
Essential Understanding You can find the area of a circle when you know its radius. You can use the area of a circle to find the area of part of a circle formed by two radii and the arc the radii form when they intersect with the circle.

take note

Theorem 10-11 Area of a Circle

The area of a circle is the product of π and the square of the radius.

$$A = \pi r^2$$



Problem 1 Finding the Area of a Circle

Think

What do you need in order to use the area formula?
You need the radius. The diameter is given, so you can find the radius by dividing the diameter by 2.

Sports **What is the area of the circular region on the wrestling mat?**

Since the diameter of the region is 32 ft, the radius is $\frac{32}{2}$, or 16 ft.

$A = \pi r^2$ Use the area formula.

$= \pi(16)^2$ Substitute 16 for r.

$= 256\pi$ Simplify.

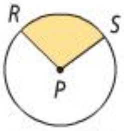
≈ 804.2477193 Use a calculator.

The area of the wrestling region is about 804 ft^2.

Got It? **1. a.** What is the area of a circular wrestling region with a 42-ft diameter?

b. **Reasoning** If the radius of a circle is halved, how does its area change? Explain.

A **sector of a circle** is a region bounded by an arc of the circle and the two radii to the arc's endpoints. You name a sector using one arc endpoint, the center of the circle, and the other arc endpoint.



Sector *RPS*

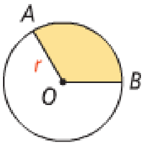
The area of a sector is a fractional part of the area of a circle. The area of a sector formed by a 60° arc is $\frac{60}{360}$, or $\frac{1}{6}$, of the area of the circle.

take note

Theorem 10-12 Area of a Sector of a Circle

The area of a sector of a circle is the product of the ratio $\frac{\text{measure of the arc}}{360}$ and the area of the circle.

$$\text{Area of sector } AOB = \frac{m\overset{\frown}{AB}}{360} \cdot \pi r^2$$

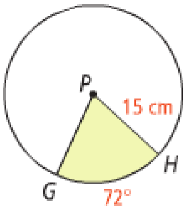


Problem 2 Finding the Area of a Sector of a Circle

Think

What fraction of a circle's area is the area of a sector formed by a 72° arc?
The area of a sector formed by a 72° arc is $\frac{72}{360}$, or $\frac{1}{5}$, of the area of the circle.

What is the area of sector *GPH*? Leave your answer in terms of π.



$\text{area of sector } GPH = \frac{m\overset{\frown}{GH}}{360} \cdot \pi r^2$

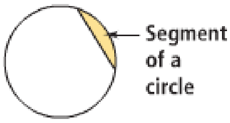
$= \frac{72}{360} \cdot \pi(15)^2$ Substitute 72 for $m\overset{\frown}{GH}$ and 15 for r.

$= 45\pi$ Simplify.

The area of sector GPH is 45π cm^2.

Got It? **2.** A circle has a radius of 4 in. What is the area of a sector bounded by a 45° minor arc? Leave your answer in terms of π.

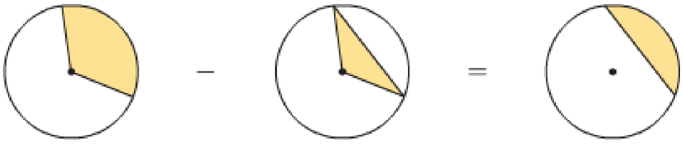
A part of a circle bounded by an arc and the segment joining its endpoints is a **segment of a circle.**



To find the area of a segment for a minor arc, draw radii to form a sector. The area of the segment equals the area of the sector minus the area of the triangle formed.

take note

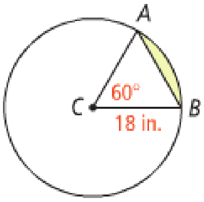
Key Concept Area of a Segment



Area of sector − Area of triangle = Area of segment

Problem 3 Finding the Area of a Segment of a Circle

What is the area of the shaded segment shown at the right? Round your answer to the nearest tenth.



Know

- The radius and $m\widehat{AB}$
- $\overline{CA} \cong \overline{CB}$ and $m\angle ACB$

Need

The area of sector ACB and the area of $\triangle ACB$

Plan

Subtract the area of $\triangle ACB$ from the area of sector ACB.

area of sector $ACB = \dfrac{m\widehat{AB}}{360} \cdot \pi r^2$ Use the formula for area of a sector.

$= \dfrac{60}{360} \cdot \pi(18)^2$ Substitute 60 for $m\widehat{AB}$ and 18 for r.

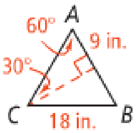
$= 54\pi$ Simplify.

Think

What kind of triangle is $\triangle ACB$?

Since $\overline{CA} \cong \overline{CB}$, the base angles of $\triangle ACB$ are congruent. By the Triangle-Angle-Sum Theorem, $m\angle A = m\angle B = 60$. So, $\triangle ACB$ is equiangular, and therefore equilateral.

$\triangle ACB$ is equilateral. The altitude forms a 30°-60°-90° triangle.



area of $\triangle ACB = \frac{1}{2}bh$ Use the formula for area of a triangle.

$= \frac{1}{2}(18)(9\sqrt{3})$ Substitute 18 for b and $9\sqrt{3}$ for h.

$= 81\sqrt{3}$ Simplify.

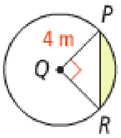
area of shaded segment $=$ area of sector ACB $-$ area of $\triangle ACB$

$= 54\pi - 81\sqrt{3}$ Substitute.

≈ 29.34988788 Use a calculator.

The area of the shaded segment is about 29.3 in.^{2}.

Got It? 3. What is the area of the shaded segment shown at the right? Round your answer to the nearest tenth.



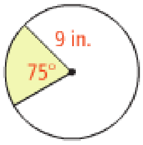
Lesson Check

Do you know HOW?

1. What is the area of a circle with diameter 16 in.? Leave your answer in terms of π.

Find the area of the shaded region of the circle. Leave your answer in terms of π.

2.



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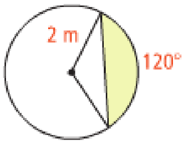
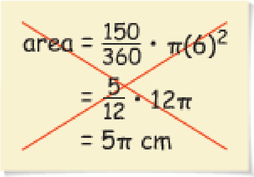


Do you UNDERSTAND?

4. **Vocabulary** What is the difference between a sector of a circle and a segment of a circle?

5. **Reasoning** Suppose a sector of $\odot P$ has the same area as a sector of $\odot O$. Can you conclude that $\odot P$ and $\odot O$ have the same area? Explain.

6. **Error Analysis** Your class must find the area of a sector of a circle determined by a 150° arc. The radius of the circle is 6 cm. What is your classmate's error? Explain.



$$\text{area} = \frac{150}{360} \cdot \pi(6)^2$$
$$= \frac{5}{12} \cdot 12\pi$$
$$= 5\pi \text{ cm}$$

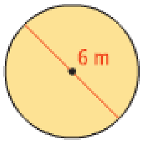
Practice and Problem-Solving Exercises

A Practice

Find the area of each circle. Leave your answer in terms of π.

See Problem 1.

7.



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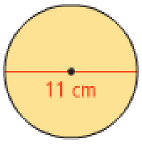

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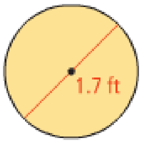

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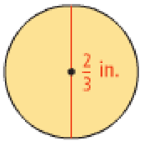


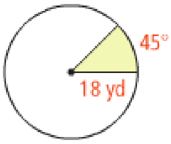
11. **Agriculture** Some farmers use a circular irrigation method. An irrigation arm acts as the radius of an irrigation circle. How much land is covered with an irrigation arm of 300 ft?

12. You use an online store locator to search for a store within a 5-mi radius of your home. What is the area of your search region?

Find the area of each shaded sector of a circle. Leave your answer in terms of π.

See Problem 2.

13.



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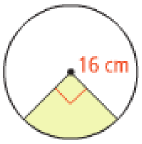

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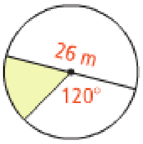

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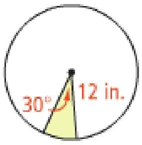

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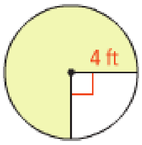

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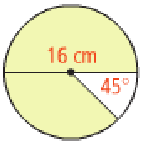


Find the area of sector TOP in $\odot O$ using the given information. Leave your answer in terms of π.

19. $r = 5$ m, $m\widehat{TP} = 90$

20. $r = 6$ ft, $m\widehat{TP} = 15$

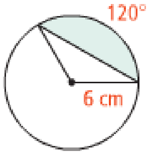
21. $d = 16$ in., $m\widehat{PT} = 135$

22. $d = 15$ cm, $m\widehat{POT} = 180$

Find the area of each shaded segment. Round your answer to the nearest tenth.

See Problem 3.

23.



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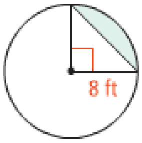

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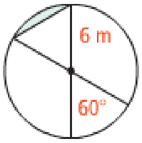
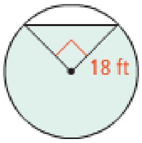


Find the area of the shaded region. Leave your answer in terms of π and in simplest radical form.

B Apply

26.



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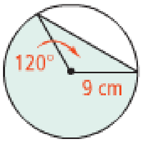

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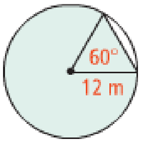

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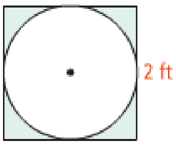

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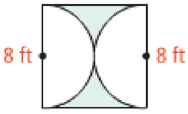

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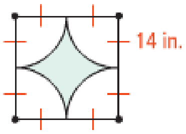
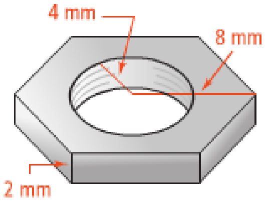


32. Transportation A town provides bus transportation to students living beyond 2 mi of the high school. What area of the town does *not* have the bus service? Round to the nearest tenth.

33. Design A homeowner wants to build a circular patio. If the diameter of the patio is 20 ft, what is its area to the nearest whole number?

34. Think About a Plan A circular mirror is 24 in. wide and has a 4-in. frame around it. What is the area of the frame?

- How can you *draw a diagram* to help solve the problem?
- What part of a circle is the width?
- Is there more than one area to consider?



35. Industrial Design Refer to the diagram of the regular hexagonal nut. What is the area of the hexagonal face to the nearest millimeter?

36. Reasoning $\overline{AB}$ and $\overline{CD}$ are diameters of $\odot O$. Is the area of sector AOC equal to the area of sector BOD? Explain.

37. A circle with radius 12 mm is divided into 20 sectors of equal area. What is the area of one sector to the nearest tenth?

38. The circumference of a circle is 26π in. What is its area? Leave your answer in terms of π.

39. In a circle, a 90° sector has area 36π in.^{2}. What is the radius of the circle?

40. **Open-Ended** Draw a circle and a sector so that the area of the sector is 16π cm^2. Give the radius of the circle and the measure of the sector's arc.

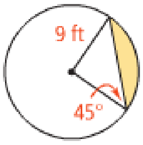
41. A method for finding the area of a segment determined by a minor arc is described in this lesson.

a. **Writing** Describe two ways to find the area of a segment determined by a major arc.

b. If $m\widehat{AB} = 90$ in a circle of radius 10 in., find the areas of the two segments determined by $\widehat{AB}$.

Find the area of the shaded segment to the nearest tenth.

42.



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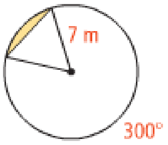

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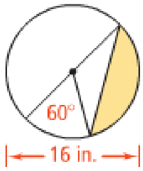
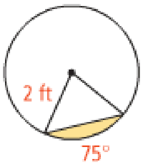


Challenge **Find the area of the shaded region. Leave your answer in terms of π.**

45.



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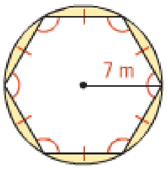

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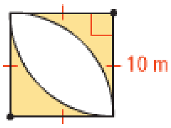
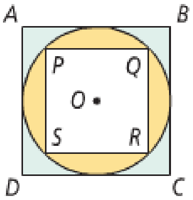


48. **Recreation** An 8 ft-by-10 ft floating dock is anchored in the middle of a pond. The bow of a canoe is tied to a corner of the dock with a 10-ft rope, as shown in the picture below.

a. Sketch a diagram of the region in which the bow of the canoe can travel.

b. What is the area of that region? Round your answer to the nearest square foot.

49. ⊙O at the right is inscribed in square $ABCD$ and circumscribed about square $PQRS$. Which is smaller, the blue region or the yellow region? Explain.



50. Circles T and U each have radius 10 and $TU = 10$. Find the area of the region that is contained inside both circles. (*Hint:* Think about where T and U must lie in a diagram of ⊙T and ⊙U.)

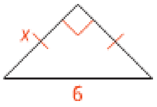
Standardized Test Prep

SAT/ACT

51. A circular tabletop has a diameter of 6 ft. What is its area?

Ⓐ 6π ft^2 Ⓑ 9π ft^2 Ⓒ 12π ft^2 Ⓓ 36π ft^2

52. What is the value of x in the diagram at the right?



Ⓕ 3 Ⓗ $6\sqrt{2}$

Ⓖ $3\sqrt{2}$ Ⓘ 9

53. The radius of ⊙P is 3 cm and the measure of central $\angle APB$ is 100. What is the measure of $\widehat{AB}$?

Ⓐ 50 Ⓑ 100 Ⓒ 260 Ⓓ 300

Extended Response

54. A circle has area 81π yd^2.

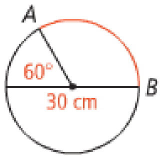
a. What is the circumference of the circle?

b. What is the length of a 45° arc of this circle? Show all your work.

Mixed Review

Find the length of $\widehat{AB}$ in each circle. Leave your answers in terms of π. **See Lesson 10-6.**

55.



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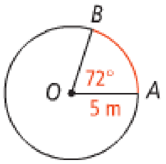

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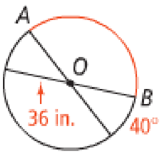


58. Three sides of a trapezoid are congruent. The fourth side is 4 in. longer than each of the other three. The perimeter is 49 in. What is the length of each side? **See Lesson 6-6.**

Get Ready! **To prepare for Lesson 10-8, do Exercises 59–63.**

59. ⊙A has radius 4 cm and ⊙B has radius 6 cm. What is the ratio of the area of ⊙A to the area of ⊙B? Write your answer in simplest form. **See Lesson 10-7.**

You roll a number cube. Find each probability. **See p. 838.**

60. $P(4)$ **61.** $P(2 \text{ or } 5)$ **62.** $P(\text{odd number})$ **63.** $P(\text{prime number})$

Concept Byte

Use With Lesson 10-7

TECHNOLOGY

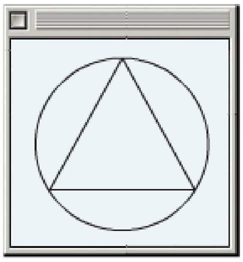
Exploring Area and Circumference

A polygon is *inscribed* in a circle when all its vertices are on the circle. Work in small groups to investigate inscribed regular polygons. Record your results in a table like the one below.

Regular Polygon			Circle		Ratios	
Sides	Perimeter	Area	Circumference	Area	$\frac{\text{Perimeter}}{\text{Circumference}}$	$\frac{\text{Polygon Area}}{\text{Circle Area}}$
3	■	■	■	■	■	■

Construct

Step 1 Use geometry software to construct a circle. Find its circumference and area. Record them in your table.



Step 2 Inscribe an equilateral triangle in the circle. Your software may do this automatically. If not, select three points that are approximately evenly spaced around the circle. Draw a triangle with these points as vertices.

Investigate

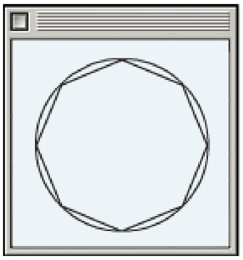
Step 3 Use your geometry software to find the perimeter and area of the triangle. Record them in your table. Calculate and record the ratios $\frac{\text{triangle perimeter}}{\text{circle circumference}}$ and $\frac{\text{triangle area}}{\text{circle area}}$.

Step 4 Change the size of the circle. Do the ratios you calculate stay the same or do they change?

Step 5 Repeat Steps 3 and 4 for a square and a regular pentagon.

Exercises

1. **Make a Conjecture** What will happen to the ratios as you increase the number of sides of the polygon? As you increase the size of the circle?



Regular Octagon

Extend

2. Extend your table to include a regular 12-sided polygon.
   - **a.** Do your conjectures hold for the 12-sided polygon?
   - **b.** Compare the ratios $\frac{\text{perimeter}}{\text{circumference}}$ and $\frac{\text{polygon area}}{\text{circle area}}$ in your table. How do they differ?
3. **Estimation** Estimate the perimeter and area of a polygon of 100 sides that is inscribed in a circle with a radius of 10 cm. Explain how you made your estimate.

10 Chapter Review

Connecting BIG ideas and Answering the Essential Questions

1 Measurement
You can find the area of a polygon, or the circumference or area of a circle, by first determining which formula to use. Then you can substitute the needed measures into the formula.

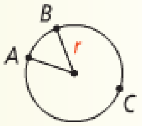
2 Similarity
The perimeters of similar polygons are proportional to the ratio of corresponding measures. The areas are proportional to the squares of corresponding measures.

Areas of Polygons (Lessons 10-1, 10-2, and 10-3)

Parallelogram $A = bh$
Triangle $A = \frac{1}{2}bh$
Trapezoid $A = \frac{1}{2}h(b_1 + b_2)$
Rhombus or kite $A = \frac{1}{2}d_1d_2$
Regular polygon $A = \frac{1}{2}ap$

Circles and Arcs (Lesson 10-6)

$C = \pi d$ or $C = 2\pi r$
$m\overparen{ABC} = m\overparen{AB} + m\overparen{BC}$
length of $\overparen{AB} = \frac{m\overparen{AB}}{360} \cdot 2\pi r$

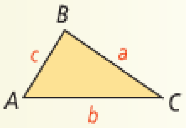


Perimeter and Area (Lesson 10-4)

If the scale factor of two similar figures is $\frac{a}{b}$, then
(1) the ratio of their perimeters is $\frac{a}{b}$ and
(2) the ratio of their areas is $\frac{a^2}{b^2}$.

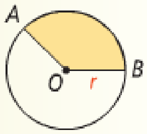
Area of a Triangle Given SAS (Lesson 10-5)

Area of $\triangle ABC = \frac{1}{2}bc(\sin A)$



Areas of Circles and Sectors (Lesson 10-7)

Area of $\odot O = \pi r^2$
Area of sector AOB
$= \frac{m\overparen{AB}}{360} \cdot \pi r^2$



Chapter Vocabulary

- adjacent arcs (p. 650)
- altitude (p. 616)
- apothem (p. 629)
- arc length (p. 653)
- base (pp. 616, 618)
- central angle (p. 649)
- circle (p. 649)
- circumference (p. 651)
- concentric circles (p. 651)
- congruent arcs (p. 653)
- congruent circles (p. 649)
- diameter (p. 649)
- geometric probability (p. 668)
- height (pp. 616, 618, 623)
- major arc (p. 649)
- minor arc (p. 649)
- radius (pp. 629, 649)
- sector of a circle (p. 661)
- segment of a circle (p. 662)
- semicircle (p. 649)

Choose the correct term to complete each sentence.

1. You can use any side as the __?__ of a triangle.

2. A(n) __?__ is a region bounded by an arc and the two radii to the arc's endpoints.

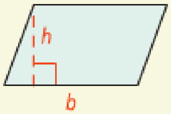
3. The distance from the center to a vertex is the __?__ of a regular polygon.

4. Two arcs of a circle with exactly one point in common are __?__.

10-1 Areas of Parallelograms and Triangles

Quick Review

You can find the area of a rectangle, a parallelogram, or a triangle if you know the **base** b and the **height** h.

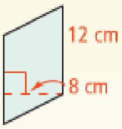


The area of a rectangle or parallelogram is $A = bh$.

The area of a triangle is $A = \frac{1}{2}bh$.

Example

What is area of the parallelogram?



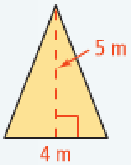
$A = bh$	Use the area formula.
$= (12)(8) = 96$	Substitute and simplify.

The area of the parallelogram is 96 cm^2.

Exercises

Find the area of each figure.

5.



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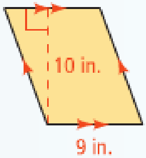

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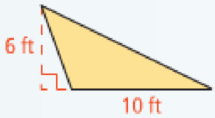

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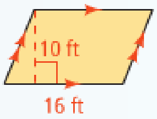
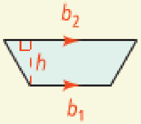


9. A right triangle has legs measuring 5 ft and 12 ft, and hypotenuse measuring 13 ft. What is its area?

10-2 Areas of Trapezoids, Rhombuses, and Kites

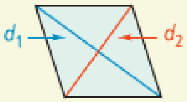
Quick Review

The **height of a trapezoid** h is the perpendicular distance between the bases, b_1 and b_2.



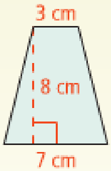
The area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$.

The area of a rhombus or a kite is $A = \frac{1}{2}d_1d_2$, where d_1 and d_2 are the lengths of its diagonals.



Example

What is the area of the trapezoid?



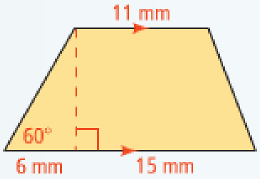
$A = \frac{1}{2}h(b_1 + b_2)$	Use the area formula.
$= \frac{1}{2}(8)(7 + 3)$	Substitute.
$= 40$	Simplify.

The area of the trapezoid is 40 cm^2.

Exercises

Find the area of each figure. If necessary, leave your answer in simplest radical form.

10.



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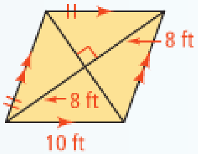

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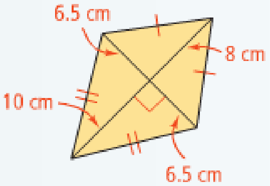

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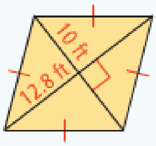
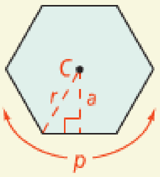


14. A trapezoid has a height of 6 m. The length of one base is three times the length of the other base. The sum of the base lengths is 18 m. What is the area of the trapezoid?

10-3 Areas of Regular Polygons

Quick Review



The **center of a regular polygon** C is the center of its circumscribed circle. The **radius** r is the distance from the center to a vertex. The **apothem** a is the perpendicular distance from the center to a side. The area of a regular polygon with apothem a and perimeter p is $A = \frac{1}{2}ap$.

Example

What is the area of a hexagon with apothem 17.3 mm and perimeter 120 mm?

$A = \frac{1}{2}ap$ Use the area formula.

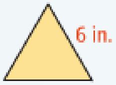
$= \frac{1}{2}(17.3)(120) = 1038$ Substitute and simplify.

The area of the hexagon is 1038 mm^2.

Exercises

Find the area of each regular polygon. If your answer is not an integer, leave it in simplest radical form.

15.



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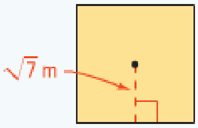


17. What is the area of a regular hexagon with a perimeter of 240 cm?

18. What is the area of a square with radius 7.5 m?

Sketch each regular polygon with the given radius. Then find its area to the nearest tenth.

19. triangle; radius 4 in.

20. square; radius 8 mm

21. hexagon; radius 7 cm

10-4 Perimeters and Areas of Similar Figures

Quick Review

If the scale factor of two similar figures is $\frac{a}{b}$, then the ratio of their perimeters is $\frac{a}{b}$, and the ratio of their areas is $\frac{a^2}{b^2}$.

Example

If the ratio of the areas of two similar figures is $\frac{4}{9}$, what is the ratio of their perimeters?

Find the scale factor.

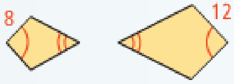
$\frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$ Take the square root of the ratio of areas.

The ratio of the perimeters is the same as the ratio of corresponding sides, $\frac{2}{3}$.

Exercises

For each pair of similar figures, find the ratio of the area of the first figure to the area of the second.

22.



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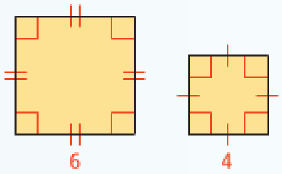

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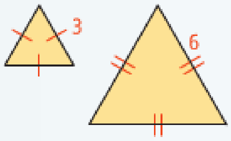

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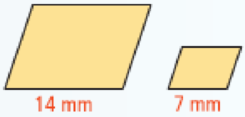


26. If the ratio of the areas of two similar hexagons is 8 : 25, what is the ratio of their perimeters?

10-5 Trigonometry and Area

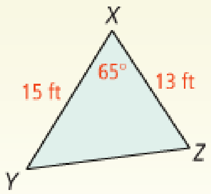
Quick Review

You can use trigonometry to find the areas of regular polygons. You can also use trigonometry to find the area of a triangle when you know the lengths of two sides and the measure of the included angle.

Area of a $\triangle$
$= \frac{1}{2} \cdot$ side length $\cdot$ side length $\cdot$ sine of included angle

Example

What is the area of $\triangle XYZ$?



$$\begin{aligned}\text{Area} &= \tfrac{1}{2} \cdot XY \cdot XZ \cdot \sin X \\ &= \tfrac{1}{2} \cdot 15 \cdot 13 \cdot \sin 65^\circ \\ &\approx 88.36500924\end{aligned}$$

The area of $\triangle XYZ$ is approximately 88 ft^2.

Exercises

Find the area of each polygon. Round your answers to the nearest tenth.

27. regular decagon with radius 5 ft

28. regular pentagon with apothem 8 cm

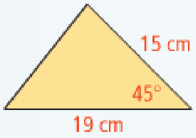
29. regular hexagon with apothem 6 in.

30. regular quadrilateral with radius 2 m

31. regular octagon with apothem 10 ft

32. regular heptagon with radius 3 ft

33.



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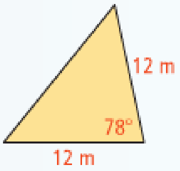


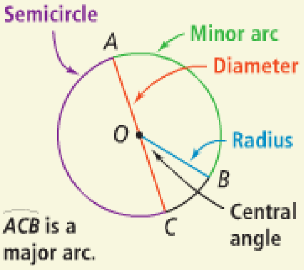
10-6 Circles and Arcs

Quick Review

A **circle** is the set of all points in a plane equidistant from a point called the **center.**

The **circumference** of a circle is $C = \pi d$ or $C = 2\pi r$.

Arc length is a fraction of a circle's circumference. The length of $\widehat{AB} = \frac{m\widehat{AB}}{360} \cdot 2\pi r$.



Example

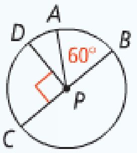
A circle has a radius of 5 cm. What is the length of an arc measuring 80°?

length of $\widehat{AB} = \frac{m\widehat{AB}}{360} \cdot 2\pi r$	Use the arc length formula.
$= \frac{80}{360} \cdot 2\pi(5)$	Substitute.
$= \frac{20}{9}\pi$	Simplify.

The length of the arc is $\frac{20}{9}\pi$ cm.

Exercises

Find each measure.



35. $m\angle APD$

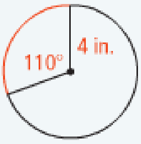
36. $m\widehat{AC}$

37. $m\widehat{ABD}$

38. $m\angle CPA$

Find the length of each arc shown in red. Leave your answer in terms of π.

39.



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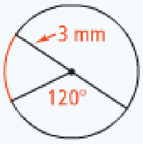

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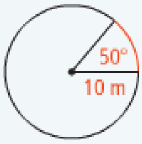

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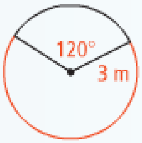
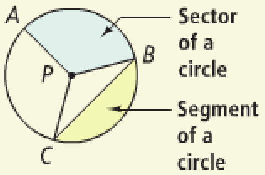


10-7 Areas of Circles and Sectors

Quick Review

The area of a circle is $A = \pi r^2$.

A **sector of a circle** is a region bounded by two radii and their intercepted arc. The area of sector $APB = \frac{m\overset{\frown}{AB}}{360} \cdot \pi r^2$.

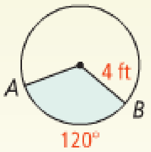


A **segment of a circle** is the part bounded by an arc and the segment joining its endpoints.

Example

What is the area of the shaded region?

$$\begin{aligned} \text{Area} &= \frac{m\overset{\frown}{AB}}{360} \cdot \pi r^2 && \text{Use the area formula.} \\ &= \frac{120}{360} \cdot \pi(4)^2 && \text{Substitute.} \\ &= \frac{16\pi}{3} && \text{Simplify.} \end{aligned}$$

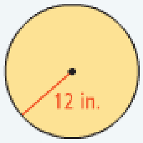


The area of the shaded region is $\frac{16\pi}{3}$ ft^2.

Exercises

What is the area of each circle? Leave your answer in terms of π.

43.



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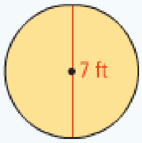
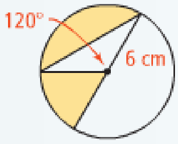


Find the area of each shaded region. Round your answer to the nearest tenth.

45.



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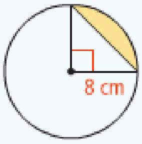


47. A circle has a radius of 20 cm. What is the area of the smaller segment of the circle formed by a 60° arc? Round to the nearest tenth.

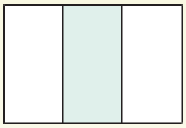
10-8 Geometric Probability

Quick Review

Geometric probability uses geometric figures to represent occurrences of events. You can use a segment model or an area model. Compare the part that represents favorable outcomes to the whole, which represents all outcomes.

Example

A ball hits the target at a random point. What is the probability that it lands in the shaded region?

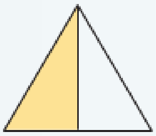


Since $\frac{1}{3}$ of the target is shaded, the probability that the ball hits the shaded region is $\frac{1}{3}$.

Exercises

A dart hits each dartboard at a random point. Find the probability that it lands in the shaded region.

48.



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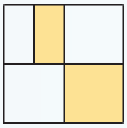

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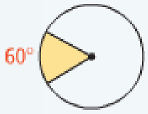

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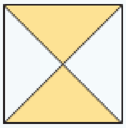

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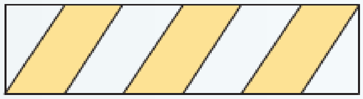

10 Chapter Test

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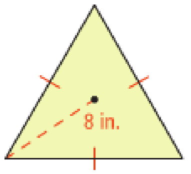
Do you know HOW?

Find the area of each figure. Round to the nearest tenth.

1.



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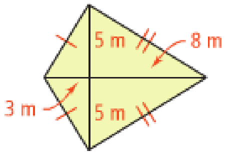
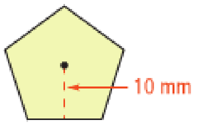


Find the area of each regular polygon. Round to the nearest tenth.

3.



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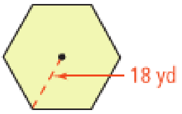
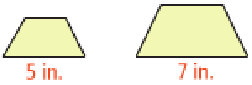


For each pair of similar figures, find the ratio of the area of the first figure to the area of the second.

5.



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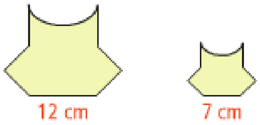
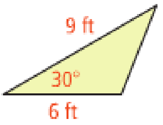


Find the area of each polygon to the nearest tenth.

7.



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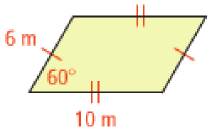


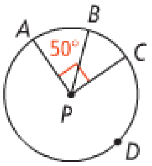
Find each measure for $\odot P$.

9. $m\angle BPC$

10. $m\widehat{AB}$

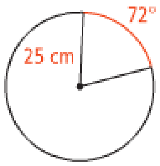
11. $m\widehat{ADC}$

12. $m\widehat{ADB}$

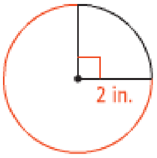


Find the length of each arc shown in red. Leave your answer in terms of π.

13.

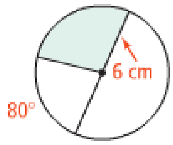


14.

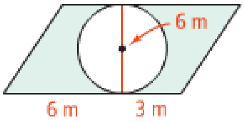


Find the area of each shaded region to the nearest hundredth.

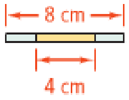
15.

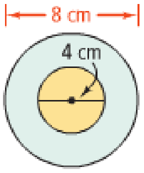


16.



17. Probability Fly A lands on the edge of the ruler at a random point. Fly B lands on the surface of the target at a random point. Which fly is more likely to land in a yellow region? Explain.





Do you UNDERSTAND?

18. Reasoning A right triangle has height 7 cm and base 4 cm. Find its area using the formulas $A = \frac{1}{2}bh$ and $A = \frac{1}{2}ab(\sin C)$. Are the results the same? Explain.

19. Mental Math Garden A has the shape of a quarter circle with radius 28 ft. Garden B has the shape of a circle with radius 14 ft. What is the ratio of the area of Garden A to the area of Garden B? Explain how you can find the answer without calculating the areas.

20. Reasoning Can a regular polygon have an apothem and a radius of the same length? Explain.

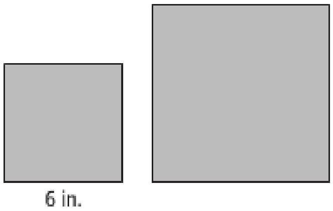
21. Open-Ended Use a compass to draw a circle. Shade a sector of the circle and find its area.

10 Cumulative Test Prep

TIPS FOR SUCCESS

Some test questions require you to relate changes in lengths to changes in areas of similar figures. Read the sample question at the right. Then follow the tips to answer it.

The ratio of the areas of the two squares shown below is 4 : 9. What is the length of a side of the larger square?



Ⓐ 4 in.

Ⓑ 9 in.

Ⓒ 13.5 in.

Ⓓ 36 in.

TIP 1

The larger square must have a side length greater than that of the smaller square. You can eliminate any answer choices that are less than or equal to 6 in.

TIP 2

All squares are similar. The ratio of their areas is equal to the square of the ratio of their side lengths.

Think It Through

The ratio of the areas is 4 : 9. The ratio of the side lengths is $\sqrt{4} : \sqrt{9}$, or 2 : 3.

Use a proportion to find the length s of the larger square.

$$\frac{2}{3} = \frac{6}{s}$$

$$s = 9$$

The correct answer is B.

Vocabulary Builder

As you solve test items, you must understand the meanings of mathematical terms. Match each term with its mathematical meaning.

A. perimeter

B. segment of a circle

C. sector of a circle

D. area

I. the part of a circle bounded by an arc and the segment joining its endpoints

II. the region of a circle bounded by two radii and their intercepted arc

III. the number of square units enclosed by a figure

IV. the distance around a figure

Multiple Choice

Read each question. Then write the letter of the correct answer on your paper.

1. What is the exact area of an equilateral triangle with sides of length 10 m?

Ⓐ $25\sqrt{3}\ \text{m}^2$

Ⓑ $25\ \text{m}^2$

Ⓒ $10\sqrt{3}\ \text{m}^2$

Ⓓ $5\sqrt{3}\ \text{m}^2$

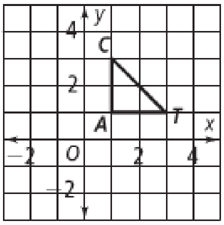
2. If $\triangle CAT$ is rotated 90° around vertex C, what are the coordinates of A'?

Ⓕ $(1, 5)$

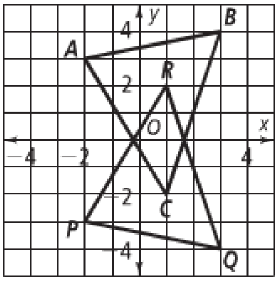
Ⓖ $(3, 3)$

Ⓗ $(-1, 3)$

Ⓘ $(3, 5)$



3. The vertices of $\triangle ABC$ have coordinates $A(-2, 3)$, $B(3, 4)$, and $C(1, -2)$. The vertices of $\triangle PQR$ have coordinates $P(-2, -3)$, $Q(3, -4)$, and $R(1, 2)$. Which transformation can you use to justify that $\triangle ABC$ is congruent to $\triangle PQR$?



Ⓐ a reflection across the y-axis

Ⓑ a reflection across the x-axis

Ⓒ a rotation 180° clockwise around the origin

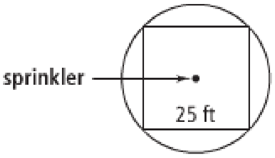
Ⓓ a translation 5 units left and 6 units down

4. If a truck's tire has a radius of 2 ft, what is the circumference of the tire in feet?

Ⓕ 8π Ⓗ 4π

Ⓖ 6π Ⓘ 2π

5. Your neighbor has a square garden, as shown below. He wants to install a sprinkler in the center of the garden so that the water sprays only as far as the corners of the garden. What is the approximate radius at which your neighbor should set the sprinkler?



Ⓐ 12.5 ft Ⓒ 25 ft

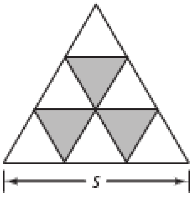
Ⓑ 18 ft Ⓓ 35 ft

6. Which of the following could be the side lengths of a right triangle?

Ⓕ 4.1, 6.2, 7.3 Ⓗ 3.2, 5.4, 6.2

Ⓖ 40, 60, 72 Ⓘ 33, 56, 65

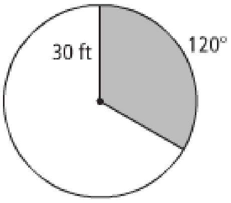
7. Every triangle in the figure at the right is an equilateral triangle. What is the total area of the shaded triangles?



Ⓐ $\frac{s^2\sqrt{3}}{36}$ Ⓒ $\frac{s^2\sqrt{3}}{9}$

Ⓑ $\frac{s^2\sqrt{3}}{12}$ Ⓓ $\frac{s^2\sqrt{3}}{6}$

8. The shaded part of the circle below represents the portion of a garden where a landscaper planted rose bushes. What is the approximate area of the part of the garden used for rose bushes?



Ⓕ 300 ft^2 Ⓗ 3769 ft^2

Ⓖ 942 ft^2 Ⓘ 8482 ft^2

9. The vertices of $\triangle ART$ are $A(1, -2)$, $R(5, -1)$, and $T(4, -4)$. If the triangle is translated 2 units to the left and 5 units up, what are the coordinates of the image triangle $A'R'T'$?

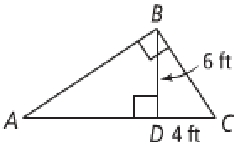
Ⓐ $A'(5, -1)$, $R'(4, -4)$, $T'(1, -2)$

Ⓑ $A'(6, -4)$, $R'(10, -3)$, $T'(9, -6)$

Ⓒ $A'(3, 3)$, $R'(7, 4)$, $T'(6, 1)$

Ⓓ $A'(-1, 3)$, $R'(3, 4)$, $T'(2, 1)$

10. In the figure below, what is the length of $\overline{AD}$?



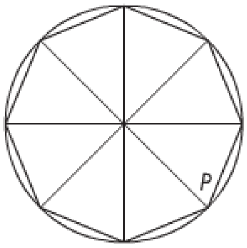
Ⓕ $2\sqrt{13}$ ft Ⓗ 10 ft

Ⓖ 9 ft Ⓘ $3\sqrt{13}$ ft

GRIDDED RESPONSE

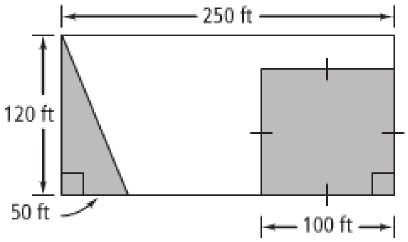
11. A triangle has a perimeter of 81 in. If you divide the length of each side by 3, what is the perimeter, in inches, of the new triangle?

12. A jewelry maker designed a pendant like the one shown below. It is a regular octagon set in a circle. Opposite vertices are connected by line segments. What is the measure of angle *P* in degrees?



13. The diagonal of a rectangular patio makes a 70° angle with a side of the patio that is 60 ft long. What is the area, to the nearest square foot, of the patio?

14. What is the area, in square feet, of the unshaded part of the rectangle below?



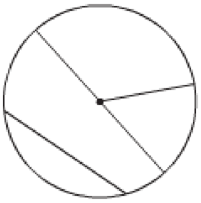
15. You are making a scale model of a building. The front of the actual building is 60 ft wide and 100 ft tall. The front of your model is 3 ft wide and 5 ft tall. What is the scale factor of the reduction?

16. At 8 o'clock, what is the measure, in degrees, of the angle formed by the two hands of the clock?

17. A square has an area of 225 cm^2. If you double the length of each side, what is the area, in square centimeters, of the new square?

Short Response

18. Use a compass and straightedge to copy the diagram of the circle below. Label the center, radius, and diameter. Then label a central angle, an arc, a sector, and a segment.



19. The coordinates of $\triangle ABC$ are $A(2, 3)$, $B(10, 9)$, and $C(10, -3)$. Is $\triangle ABC$ an equilateral triangle? Explain.

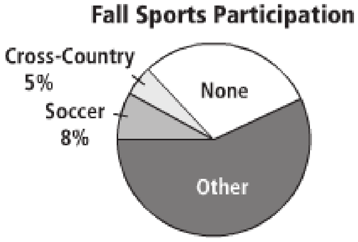
20. Draw a right triangle. Then construct a second triangle congruent to the first. Show your steps.

21. One diagonal of a parallelogram has endpoints at $P(-2, 5)$ and $R(1, -1)$. The other diagonal has endpoints at $Q(1, 5)$ and $S(-2, -1)$. What type of parallelogram is *PQRS*? Explain.

Extended Response

22. The coordinates of the vertices of isosceles trapezoid *ABCD* are $A(0, 0)$, $B(6, 8)$, $C(10, 8)$, and $D(16, 0)$. The coordinates of the vertices of isosceles trapezoid *AFGH* are $A(0, 0)$, $F(3, 4)$, $G(5, 4)$, and $H(8, 0)$. Are the two trapezoids similar? Justify your answer.

23. The circle graph below shows fall sports participation at one school.



a. How many degrees greater is the measure of the angle that represents soccer than the measure of the angle that represents cross-country? Show your work.

b. If 48 students play soccer, how many run cross-country? Explain your reasoning.